MATH 70 SECTION 01 FINAL EXAM
FALL 2017

This version of the exam is for SECTION 01, taught by Professor Walsh, which meets: Tues/Thurs/Fri 9:30 Do you have the right version of the exam? This is Question 0, and is *worth five points*.

Problem #	Point Value	Points
Having the right exam	5	
1	15	
2	10	
3	10	
4	12	
5	8	
6	6	
7	9	
8	8	
9	9	
10	8	
Total	100	

(1) (5+10=15 points) Short answer questions: no partial credit, and no work or explanations required:

True or False? (circle your answers)

(a) Suppose A is a 5×3 matrix which has 3 pivots. Let T be the linear transformation defined by $T(\vec{x}) = A\vec{x}$. T is not onto. T F

(b) Every linearly independent set in $\mathbb{R}^n$ is an orthogonal set. T F

(c) If the dimension of the vector space V is p for some $p \geq 1$, then every set of vectors that spans V has more than p vectors. T F

(d) There exists a one-to-one linear transformation from $\mathbb{P}_3$ to $\mathbb{R}^3$. T F

(e) Suppose A is an $m \times n$ matrix. Then Nul$A$ is orthogonal to Col$A$. T F

Short Answer

(a) Suppose U is a square matrix with orthonormal columns. Explain why U is invertible using theorems from the class.

(b) Suppose a 8×6 matrix A has 4 pivot columns. What is the dimension of Nul$A$?

(c) Suppose W is a subspace of $\mathbb{R}^n$. If I take the union of orthogonal bases for W and $W^{\perp}$, why does this set span $\mathbb{R}^n$?

(d) Gram-Schmidt is an algorithm for doing what?

(e) Suppose A, B are both $n \times n$ matrices for some n. Show that if A is similar to B, then A^2 is similar to B^2.

Questions 2-8 have partial credit, and work/explanations/justifications ARE required:

(2) (2+4+4=10 pts) Consider the matrix A below.

$$\mathcal{A} = \begin{pmatrix} 3 & 1 \\ 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

(a) Show that the columns of A are orthogonal.

(b) Show that the vector $\vec{y} = \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix}$ is *not* in Col$A$.

(c) Find the vector $\hat{y}$ in Col$A$ that is closest to $\vec{y}$.

(3) (2+4+2=10 pts) Consider the matrix A below.

$$A = \begin{pmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & h & 3 \\ 0 & 0 & 4 & 14 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

(a) What are the 4 eigenvalues of A? (Note this does not depend on what the value of h is!)

(b) What value of h will make the eigenspace for $\lambda = 4$ two dimensional?

(c) Suppose you put this value of h in A. What would you do next to decide whether A was diagonalizable or not? In particular, what would need to be true for A to be diagonalizable?

(4) (2+3+3+4=12 pts) Let $M_{2\times 2}$ be the vector space of 2×2 real matrices with real entries. Consider the transformation $f : M_{2\times 2} \to \mathbb{R}^2$ given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{bmatrix} 3a+b \\ c+d \end{bmatrix}$$

(a) Show that f is linear.

(b) Find a matrix for the linear transformation f in terms of the basis:

$$\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\}$$

for $M_{2\times 2}$ and the standard basis $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$ for $\mathbb{R}^2$.

(c) What does it mean for a transformation T to be one-to-one?

(d) Either prove f as above is one-to-one, or find specific matrices that show it is not.

(5) (2+4+2=8 pts) Let W be the subspace with basis $\vec{v_1} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$.

(a) Verify that $\vec{v_1}$ and $\vec{v_2}$ are NOT orthogonal.

(b) Find an orthogonal basis for W by replacing $\vec{v_2}$ with vector a $\vec{u_2}$ that is orthogonal to $\vec{v_1}$ with $\text{Span}\{\vec{v_1}, \vec{u_2}\} = W$.

(c) Suppose we let $\vec{v}$ we a vector that is not in W. Explain what I would do to find a vector $\vec{u_3}$ such that $\{\vec{u_1}, \vec{u_2}, \vec{u_3}\}$ is an orthogonal basis for $\mathbb{R}^3$. Draw a schematic diagram if that helps!

(6) (6 pts) Suppose B is the reduced echelon form for the matrix A.

$$\mathcal{A} = \begin{pmatrix} 1 & 2 & -4 & 3 & 3 \\ 5 & 10 & -9 & -7 & 8 \\ 4 & 8 & -9 & -2 & 7 \\ -2 & -4 & 5 & 0 & -6 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 0 & -5 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) Find a basis for Nul A.

(b) Find a basis for Col A.

(c) Let $\vec{b} = \begin{bmatrix} 2 \\ 8 \\ 4 \\ -17 \end{bmatrix}$. Suppose $\begin{bmatrix} -1 \\ 0 \\ 3 \\ 0 \\ 5 \end{bmatrix}$ is a solution to the equation $A\vec{x} = \vec{b}$. Describe the solution set to $A\vec{x} = \vec{b}$ in parametric form.

(7) (9 pts) For each of the following give an example of a matrix with the stated property. EXPLAIN why your examples work.

(a) Find a 2×2 matrix that is invertible but not diagonalizable.

(b) Find a 2×2 matrix that is diagonalizable but not invertible.

(c) Find a 2×3 matrix A NOT in reduced echelon form such that the mapping $\vec{x} \mapsto A\vec{x}$ is *not* onto.

(8) (6+2=8 pts) Consider the matrix A given here:

$$A = \begin{pmatrix} 1 & -6 \\ 2 & -6 \end{pmatrix}$$

(a) Diagonalize the matrix A. That is, find matrices P, D with $A = PDP^{-1}$.

(b) Use your answer from the previous part to *explain how you would* compute A^{37}

(9) (2+2+6=10 pts) Suppose W is a subspace of $\mathbb{R}^n$. Consider the set $W^\perp$.

(a) What does it mean for the a vector $\vec{z}$ from $\mathbb{R}^n$ to be in $W^\perp$?

(b) What do you need to prove to show $W^\perp$ is a subspace of $\mathbb{R}^n$.

(c) Show that $W^\perp$ is a subspace of $\mathbb{R}^n$.

(10) (2+6=8 pts) Let W and U be subspaces of a vectors space V. Suppose the intersection, $W \cap U$, of W and U contains only the zero vector $\vec{0}$. Let $\{\bar{w}_1, \ldots, \bar{w}_p\}$ and $\{\bar{u}_1, \ldots, \bar{u}_k\}$ be bases of W and U, respectively.

(a) What does it mean for the set $\{\bar{w}_1, \ldots, \bar{w}_p, \bar{u}_1, \ldots, \bar{u}_k\}$ to be linearly independent - i.e. give the definition of linear independence of this set.

(b) Show that $\{\bar{w}_1, \ldots, \bar{w}_p, \bar{u}_1, \ldots, \bar{u}_k\}$ is linearly independent.

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Name: ____

I pledge that I have neither given nor received assistance on this exam.

Signature ____