Math 70	**TUFTS UNIVERSITY**	**Apr 9, 2018**
Linear Algebra	**Department of Mathematics**	**Sections 1,2,3,4**
		Exam 2

Instructions: No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. *You are required to sign your exam. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Problem	Point Value	Points
1	6	
2	6	
3	12	
4	12	
5	8	
6	7	
7	14	
8	11	
9	8	
10	16	
	100	

1. (6 pts) Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & 3 \end{bmatrix}$.

2. (6 pts) T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. Show that T is invertible:

$$T\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} -5x_1 + 9x_2 \\ 4x_1 - 7x_2 \end{pmatrix}$$

3. (12 pts) **True/false** questions. For each of the statements below, decide whether it is true or false. Indicate your answer by shading the corresponding box. There will be no partial credit.

   (a) (2 pts) If A is a 5×5 matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^5$, it is possible that for some $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution.

   T F

   (b) (2 pts) It is possible for a 4×4 matrix to be invertible when its columns do not span $\mathbb{R}^4$.

   T F

   (c) (2 pts) A square matrix with two identical columns is invertible.

   T F

   (d) (2 pts) If $A^3 = 0$ then $\det A = 0$.

   T F

   (e) (2 pts) The pivot columns of a matrix are always linearly independent.

   T F

   (f) (2 pts) If A is $m \times n$ and rank $A = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto.

   T F

4. (12 pts) Suppose that a matrix A is row equivalent to the matrix B given below:

$$B = \begin{bmatrix} 2 & 0 & 3 & -1 \\ 0 & 4 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 6 \end{bmatrix}$$

(a) (3 pts) Compute the determinant of B.

(b) (3 pts) Can you compute the determinant of A? If so, what is it? If not, why not?

Suppose that a matrix C is row equivalent to the matrix D given below:

$$D = \begin{bmatrix} 2 & 0 & 3 & -1 \\ 0 & 4 & 1 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

(c) (3 pts) Compute the determinant of D.

(d) (3 pts) Can you compute the determinant of C? If so, what is it? If not, why not?

5. (8 pts)
   (a) (4 pts) Define what it means for a subset H to be a subspace of a vector space V.

   (b) (4 pts) Let V be a vector space and let $\mathcal{B} = \{v_1, v_2, ..., v_p\}$ be an indexed set of vectors in V. Complete the following: $\mathcal{B}$ is called a basis for V if:

6. (7 pts) Prove whether each of the following sets is a subspace. **If it does form a subspace, determine its dimension.**

   (a) (3 pts) $\{p(t) \in \mathbb{P}_3 : p(2) = 2\} \subseteq \mathbb{P}_3$

   (b) (4 pts) $W = \left\{ \begin{bmatrix} 2t+2 \\ t+1 \\ 0 \\ -t-1 \end{bmatrix} : t \in \mathbb{R} \right\} \subseteq \mathbb{R}^4$

7. (14 pts) Let $T : \mathbb{P}_2 \to \mathbb{R}^2$ be the transformation defined by

$$p(t) \mapsto \begin{bmatrix} p(0) \\ p''(0) \end{bmatrix}$$

where p'' denotes the second derivative of p.

(a) (8 pts) Prove that T is a linear transformation.

(b) (6 pts) Find a basis for the kernel of T.

8. (11 pts) Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 4 & 0 & 3 \\ 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & -1 & 0 & -2 & -1 \end{bmatrix} \text{ and its row equivalent matrix } B = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) (3 pts) Find a basis for $\operatorname{Col} A$ and determine its dimension.

(b) (3 pts) Find a basis for $\operatorname{Nul} A$ and determine its dimension.

(c) (3 pts) Find a basis for $\operatorname{Row} A$ and determine its dimension.

(d) (2 pts) How many pivot columns are in a row echelon form of A^T? Explain.

9. (8 pts) Suppose a nonhomogeneous system of six equations of eight unknowns has a solution for all possible constants on the right sides of the equations. Is it possible to find two nonzero solutions of the associated homogeneous system that are not multiples of each other? Explain.

10. (16 pts) Let V be the set of all 2×2 matrices.

Let H be the subspace of V with a basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ and consider the ordered set,

$$S = \{u, v, w\} = \left\{ \begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 5 \\ 0 & 10 \end{bmatrix}, \begin{bmatrix} -2 & 10 \\ 0 & 20 \end{bmatrix} \right\}.$$

(a) (4 pts) Write $[u]_{\mathcal{B}}$, $[v]_{\mathcal{B}}$, and $[w]_{\mathcal{B}}$ in $\mathcal{B}$-coordinate vectors.

(b) (8 pts) Is $S = \{u, v, w\} = \left\{ \begin{bmatrix} -6 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 5 \\ 0 & 10 \end{bmatrix}, \begin{bmatrix} -2 & 10 \\ 0 & 20 \end{bmatrix} \right\}$ linearly independent? Explain.

(Problem 10 continues on the next page)

(Problem 10 continues)

(c) (4 pts) Find a basis for the subspace spanned by S.

Math 70 Exam II April 9th, 2018

Name: ______________________________

Circle your section:

Section 1 Michael Chou T,W,F 9:30-10:20

Section 2 Caleb Magruder M,W 3-4:15

Section 3 Mary Glaser T,Th,F 12-12:50

Section 4 Eunice Kim T,Th 3-4:15

I pledge that I have neither given nor received assistance on this exam.

Signature ______________________________