Math 46 | Tufts University
Department of Mathematics
Exam 2 | November 6, 2006

1. (10 points) Compute the determinant.

$$\begin{pmatrix} 1 & 1 & 1 \\ -3 & 8 & -4 \\ 2 & -3 & 2 \end{pmatrix}$$

2. (10 points) Let

$$A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$$

Compute $5A$. Is $\det 5A = 5 \det A$? Give reasons.

3. (10 points) If A and P are $n \times n$ matrices and P is invertible, show that $\det(PAP^{-1}) = \det A$.

4. (10 points) Find a basis for the space spanned by

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}$$

5. (10 points) Let $M_{2\times 2}$ be the space of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

and let $T : M_{2\times 2} \to \mathbf{R}^2$ be defined by $T\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \begin{pmatrix} a-b \\ 2c \end{pmatrix}$

(a) Prove that T is a linear transformation.

(b) Find a basis for $\ker T$.

Exam continues on other side.

6. (10 points) Let $\mathbf{P_2}$ be the space of polynomials $p(t)$ of degree ≤ 2 and let $B = \{1-t, 1+t, 1+t+t^2\}$ be a basis.

   (a) Find $p(t)$ if $[p(t)]_B = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$

   (b) Find $[t^2+1]_B$.

7. (2 points each) Determine which of the following are subspaces of $M_{2\times 2}$ (see exercise 5) **NO reasons, no partial credit.**

   (a) $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a = e^b\right\}$

   (b) $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a+b+c = 3\right\}$

   (c) $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a+b = c^2\right\}$

   (d) $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid \frac{a+b}{c+d} = 1\right\}$

   (e) $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a+b = c-d\right\}$

8. (10 points)
   (a) If A is a 10×14 matrix and dim Nul $A \geq 6$, find all possible values for rank A.
   (b) If A is an 8×3 matrix, what is the smallest possible dimension for Nul A?

9. (10 points) Let $T : \mathbf{R^{10}} \to \mathbf{R^6}$ be a linear transformation.
   (a) What is the maximum value for the dimension of $\ker T$? Explain.
   (b) What is the maximum value for the dimension of range T? Explain.

10. (10 points) Prove: if $T : V \to W$ is linear, then $\ker T$ is a subspace of V.

End of Exam