

**Instructions:** No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. *You are required to sign your exam. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Problem	Point Value	Points
1	10	
2	10	
3	10	
4	14	
5	12	
6	4	
7	8	
8	16	
9	8	
10	8	
	100	

1. (10 pts) **True/false questions.** For each of the statements below, decide whether it is true or false. Indicate your answer by shading the corresponding box. There will be no partial credit.

(a) Every homogeneous system of linear equations is consistent.

T  F

(b) Every homogeneous system of three linear equations in five unknowns has a nontrivial solution.

T  F

(c) Every set of three vectors in  $\mathbb{R}^5$  is linearly dependent.

T  F

(d) Every linear transformation from  $\mathbb{R}^3$  to  $\mathbb{R}^5$  is one-to-one.

T  F

(e) If a collection  $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$  is linearly dependent, then one of the vectors is a scalar multiple of one of the others.

T  F

2. (10 points)

(a) Write the augmented matrix of the following linear system and row reduce it to echelon form.

$$2x_1 + 10x_2 = -6$$

$$hx_1 + 20x_2 = k$$

(b) Find all value(s) of  $h$  and  $k$  so the linear system above has

(i) no solutions

(ii) only one solution

(iii) infinitely many solutions

(c) For which values of  $h$  is the associated matrix of coefficients invertible?

3. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Is  $A$  in reduced row-echelon form? If not, give the reduced row-echelon matrix which is row equivalent to  $A$ .

(b) Let

$$\mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}.$$

Is the vector equation  $A\mathbf{x} = \mathbf{b}$  consistent? If it is, give an explicit solution. If not, explain why not.

(c) Let the mapping  $T : \mathbb{R}^5 \rightarrow \mathbb{R}^3$  be defined by  $T(\mathbf{x}) = A\mathbf{x}$ . Does  $T$  map  $\mathbb{R}^5$  onto  $\mathbb{R}^3$ ? Justify your answer.

4. (14 points) Suppose that the matrix  $A = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \ \mathbf{a}_4 \ \mathbf{a}_5]$  is row equivalent to the matrix

$$B = \begin{bmatrix} 1 & 0 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Describe all solutions of  $A\mathbf{x} = \mathbf{0}$  in parametric vector form.

(b) Write the *definition* of what it means for the columns of  $A$  to be linearly independent.

(c) Are the columns of  $A$  linearly independent? If so, justify. If not, find a dependence relation among the columns of  $A$ .

5. (12 points)

(a) Define what it means for a transformation  $T$  to be linear.

(b) Show that  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $T(x, y) = (x + 2y, -4x)$  is linear using your definition from part (a).

(c) Show that  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by  $T(x, y) = (xy, 0)$  is *not* linear.

6. (4 points) Find the standard matrix of the linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$  defined by  $T(x, y) = (2x + y, -3x + 5y, -x, 11y)$ .

7. (8 points)

(a) Show by means of an example that it is possible to have a set of vectors which spans  $\mathbb{R}^3$  but is not linearly independent.

(b) Show by means of an example that it is possible to have a set of vectors which is linearly independent but does not span  $\mathbb{R}^3$ .

8. (16 points) Given

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -2 & 4 \end{bmatrix}; \quad B = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}.$$

Compute each of the following where possible – if NOT possible, explain why not.

(a)  $AB$

(b)  $BA$

(c)  $\mathbf{y}\mathbf{x}^T$

(d)  $\mathbf{y}\mathbf{x}$



$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & -2 & 4 \end{bmatrix}; \quad B = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}; \quad \mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}.$$

Compute each of the following where possible – if NOT possible, explain why not.

(e)  $A^T$

(f)  $A^T \mathbf{x}$

(g)  $\mathbf{y}^T A^T \mathbf{x}$

9. (8 points) Let  $A = \begin{bmatrix} 2 & -2 & -1 \\ -2 & 5 & 6 \\ 10 & -4 & 5 \end{bmatrix}$ . Find the inverse of  $A$ , if it exists.

(You are not allowed to use any results about determinants.)

10. (8 points) Let  $A$  and  $B$  be  $3 \times 3$  matrices. Suppose  $A$  is invertible, but the third column of  $B$  is a linear combination of the first two columns of  $B$ . Prove, using the definition of matrix-matrix products, that  $AB$  is not invertible.

Math 70    Exam 1    February 25, 2013

Name \_\_\_\_\_

Instructor \_\_\_\_\_

I pledge that I have neither given nor received assistance on this exam.

Signature \_\_\_\_\_