Tufts University

Math 70	**Department of Mathematics**	**February 22, 2016**
Linear Algebra		**Exam 1**

Instructions: No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. *You are required to sign your exam. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Please put your name and section number on the last page of this test.
Note there are ten problems and there are problems on every page but this page and the last page.

Problem	Point Value	Points
1	6	
2	20	
3	6	
4	8	
5	20	
6	8	
7	8	
8	8	
9	8	
10	8	
	100	

1. (6 points): Indicate by shading the appropriate box whether each statement is true or false. *For this problem you do not need to give reasons.*

   (a) Let A be a 4×3 matrix. Assume there is a unique solution to $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$. Then, there is a unique solution to $A\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 8 \end{bmatrix}$. T F

   (b) Any set of five vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$. T F

   (c) Assume A is a 5×4 matrix and assume $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Then, the columns of A span $\mathbb{R}^5$. T F

2. (20 points): Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$.

(a) Find all values of h such that the system $A\mathbf{x} = \begin{bmatrix} 2 \\ h \\ 1 \end{bmatrix}$ is consistent.

(b) For each h for which the system in (a) is consistent, how many solutions are there? Why?

3. (6 points): Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ defined for $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + 2x_3 \\ 2x_1 + 4x_2 - x_3 \\ x_2 - x_3 \end{bmatrix}.$$ Find the standard matrix A of T.

4. (8 points): Let A be a 6×6 matrix with five pivot columns.

   (a) How many solutions are there to $A\mathbf{x} = \mathbf{0}$? Why?

   (b) Is there a solution to $A\mathbf{x} = \mathbf{b}$ for all $\mathbf{b} \in \mathbb{R}^6$? Why or why not?

5. (20 points): Let T be the linear transformation with standard matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 2 & 5 \end{bmatrix}$.

(a) (2 points): What is the domain of T? ____________________

(b) (2 points): What is the codomain (target) of T?____________________

(c) Is $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$ in range(T)? Why or why not?

(d) Is T one-to-one? Why or why not?

6. (8 points): Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ be vectors in $\mathbb{R}^2$. Prove that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ *is* linearly dependent.
HINT: You may use facts about pivots or linear equations.

7. (8 points): Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a transformation. Give the definition for T to be a *linear transformation.*

8. (8 points): Let T be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Complete the following definitions

   (a) T is *one-to-one (injective)* if . . .

   (b) T is *onto (surjective)* if . . .

9. (8 points): Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be vectors in $\mathbb{R}^4$ and assume $\{\mathbf{v}_1, \mathbf{v}_2\}$ is linearly independent. Prove that the set of vectors $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2\}$ is also linearly independent.
HINT: Use the definition of linear independence and that $\{\mathbf{v}_1, \mathbf{v}_2\}$ *is linearly independent.*

10. (8 points): Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 + x_2 \\ 3x_2 \end{bmatrix}$. Prove T satisfies the conditions from the definition to be a linear transformation.

HINT: start the proof by letting $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ *and* $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ *be vectors in* $\mathbb{R}^2$ *and* $c \in \mathbb{R}$.

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Name: ______________________________

Instructor: ______________________________

Section number: ______

I pledge that I have neither given nor received assistance on this exam.

Signature ______________________________