The set of all linear combinations of the row vectors of a matrix A is called the **row space** of A and is denoted by Row A.

$$\boxed{\text{Col } A^T = \text{Row } A}.$$

THEOREM 13

If two matrices A and B are row equivalent, then their row spaces are the same. If B is in echelon form, the nonzero rows of B form a basis for the row space of A as well as B.

DEFINITION

The **rank** of A is the dimension of the column space of A.

$$\boxed{\text{rank } A = \dim \text{Col } A = \#\text{ of pivot columns of } A = \dim \text{Row } A}.$$

$$\underbrace{\text{rank } A}_{\updownarrow} + \underbrace{\text{dim Nul } A}_{\updownarrow} = \underbrace{n}_{\updownarrow}$$

$$\left\{\begin{array}{c}\#\text{ of pivot}\\ \text{columns}\\ \text{of } A\end{array}\right\} \qquad \left\{\begin{array}{c}\#\text{ of nonpivot}\\ \text{columns}\\ \text{of } A\end{array}\right\} \qquad \left\{\begin{array}{c}\#\text{ of}\\ \text{columns}\\ \text{of } A\end{array}\right\}$$

THEOREM 14 THE RANK THEOREM

The dimensions of the column space and the row space of an $m \times n$ matrix A are equal. This common dimension, the rank of A, also equals the number of pivot positions in A and satisfies the equation

$$\text{rank } A + \dim \text{Nul } A = n.$$

Since Row A = Col A^T,

$$\boxed{\text{rank } A = \text{rank } A^T}.$$

THE INVERTIBLE MATRIX THEOREM (continued)

Let A be a square $n \times n$ matrix. The the following statements are equivalent:

m. The columns of A form a basis for $\mathbf{R}^n$

n. Col $A = \mathbf{R}^n$

o. dim Col $A = n$

p. rank $A = n$

q. Nul $A = \{\mathbf{0}\}$

r. dim Nul $A = 0$