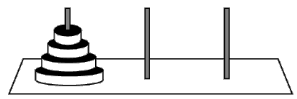
MATH 61-02: WORKSHEET 4 (§2.2-2.3)

(W1) The *Towers of Hanoi* is a game consisting of 3 rods and n disks of different sizes, looking like this:

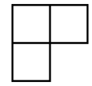


The disks start out in ascending size order on the left rod (as in the figure). The aim is to move all disks to the right rod while obeying the following rule: each move consists of taking the one top disk from one rod and moving it to another rod, while never placing a disk on top of a smaller disk. Practice with 2, 3, and 4 disks online (http://vornlocher.de/tower.html).

Prove by induction: If there are n disks, the game can be won in $2^n - 1$ moves.

(Bonus: convince yourself this is optimal.)

(W2) Suppose we place five points in a square of area 16. Prove that there exist two points which are a distance of at most $2\sqrt{2}$ apart. Can you come up with a similar statement for some number of points in a pentagon?

(W3) An *L-triomino* is an L-shaped cluster of three squares, like this: 

Prove that for all $n \in \mathbb{N}$, a $2^n \times 2^n$ checkerboard with any one square removed can be tiled with L-triominoes. (You can email kristofer.siy@tufts.edu for a hint on this one if you're stuck.)

(W4) I went to this crazy party last weekend where they managed to cram all 5200 undergraduates at Tufts into a single house. Of course, since we didn't all know each other, some people shook hands to introduce themselves. I want to prove that there were two people at the party who shook hands with the same number of people. One possible solution starts like this: *Suppose not. Let's number the people 1 through 5200, and let* $h(n)$ *denote the number of people that person* n *shook hands with. Then the numbers* $h(1), h(2), \ldots, h(5200)$ *are 5200 different integers between 0 and 5199.* Finish the proof. (There are a few more steps to go.)