

MATH 61-02: WORKSHEET 1 (§1.1-1.2)

- (W1) Let $\mathcal{P}(X)$ denote the power set of a set X . Let $A = \{3, 7\}$ and $B = \{4, 7, 12\}$.
- What is $\mathcal{P}(A \cup B)$? What is $\mathcal{P}(A) \cup \mathcal{P}(B)$? Name one element contained in one of these sets but not the other. Name one element contained in both of these sets.
 - Let S and T be arbitrary sets. When does $\mathcal{P}(S \cup T) = \mathcal{P}(S) \cup \mathcal{P}(T)$? Explain your reasoning.
- (W2) Consider the following collections of sets given by: $\mathcal{A}_n := \{(\frac{-1}{n}, \frac{1}{n}) \mid n \in \mathbb{N}\}$,
 $\mathcal{B}_n := \{[n, n + \frac{1}{n}) \mid n \in \mathbb{N}\}$, $\mathcal{C}_n := \{x \in \mathbb{R} \mid n < x, x^2 \in \mathbb{Q}\}$.
- Which sets within these three collections does the real number 5.1 belong to?
 - Find the union and intersection of the sets in \mathcal{A} . Is \mathcal{A} a nested collection?
 - Let m be a positive integer. Show that $\mathcal{B}_m \cap \mathcal{C}_m \neq \emptyset$.
- (W3) Let our universal set be $U = [6]$. Let $A = \{1, 2, 4, 5\}$, $B = \{1, 3, 5, 6\}$, $C = \{4, 5\}$, $D = \{1, 2, 6\}$,
 $E = \{2, 3, 6\}$. Use these sets and parentheses, unions, intersections, and complements to express the following sets. (It is possible to create all six.) As an example, the set $\{2, 6\}$ is equal to $D \cap E$, among other possible expressions.
- $\{1, 4, 5\}$
 - $\{2, 4\}$
 - $\{2\}$
 - $\{3, 4\}$
 - $\{1, 3, 6\}$
 - $\{1, 3, 5\}$
- (W4) Consider the set $S_k := \{x = ak \mid a \in \mathbb{N}\}$ defined for $k \in \mathbb{N}$.
- Give a qualitative description of the set S_k for a given k .
 - For what values of k is $12 \in S_k$? In general, suppose that $c \in \mathbb{N}$. For what values of k is $c \in S_k$?
 - What is $S_2 \cap S_3$? What is $S_9 \cap S_{12}$? Can you find a way to describe the set $S_m \cap S_n$ for two positive integers m and n in general? (Can you extend this to more than two positive integers?)
 - Can you find a set $T \subseteq \mathbb{N}$ such that
$$\bigcup_{t \in T} S_t = \mathbb{N} \setminus \{1\}?$$
- (W5) (a) Do you have any remaining questions about the concepts and topics covered in class so far?
(b) Do you feel like this worksheet was too easy, too difficult, too long, or too short (if you are considering the “worksheet track”)? Any suggestions?