

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Quiz name: Discrete Math Apr 20 (infinities)

1. If A and B are countably infinite sets, then a bijection exists from A to B.

True  
 False

Just write  $A=\{a_1, a_2, \dots\}$  and  $B=\{b_1, b_2, \dots\}$ ; then let  $f(a_i)=b_i$ .

2. If C and D are uncountable sets, then a bijection exists from C to D.

True  
 False

No, there are many different sizes of uncountable sets. For instance,  $2^c > c$ .

3. Every infinite set has a countably infinite subset.

True  
 False

Yes, you can just construct it by selecting elements one at a time and calling them  $a_1, a_2, a_3$ , etc.

4. The union of two countable sets might be uncountable.

True  
 False

No, a countable union of countable sets is countable. This is one of the classic "Hilbert's hotel" scenarios.

5. The intersection of two countably infinite sets must be countably infinite.

True  
 False

No, countably infinite sets can even be disjoint! (Like {evens} and {odds}).

6.  $(\aleph_0)^2 < c$  (aleph-nought squared is strictly less than c)

True  
 False

Aleph-nought squared equals aleph-nought. ( $\mathbb{Z}^2$  is countable.)

7. If A is a subset of B and B is a subset of C, then the inclusion map from A to C (the identity map  $f(a)=a$ ) exhibits that  $|A| \leq |C|$ .

True  
 False

To show that one cardinality is less than or equal to another, you can exhibit an injection between sets representing those cardinalities. That's what this map does! (Inclusions of subsets are always injections.)

8.  $2^c = c$  (2 to the c is equal to c)

True  
 False

Cantor says: the power set is always bigger than the set itself!