

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature, you are pledging that you have neither given nor received assistance on the exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

1. (10 points)

(a) Find the general solution of

$$\frac{dx^2}{dt^2} = 3t + 1$$

(b) Find the specific (particular) solution of the above equation that satisfies

$$x(0) = 2 \quad x'(0) = 3$$

2. (10 points) Solve $(D - 1)x = 3$

3. (15 points) Use the Wronskian to show that $e^t, \sin t, \cos t$ are linearly independent.

4. (15 points) Solve

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 1 & -1 & -3 & 1 \\ 2 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

5. (10 points) Solve $(D + 2)^3(D - 1)x = 0$

6. (10 points) Solve $(5D^2 + 2D + 1)x = 0$

7. (15 points) Make a simplified guess for a particular solution of

$$(D + 2)(D^2 + 1)x = te^{-2t} + \sin t$$

DO NOT SOLVE FOR THE COEFFICIENTS.

8. (15 points)

(a) Find solutions of the form $x = t^\alpha$ for

$$(tD^2 - D)x = 0, \quad t > 0$$

(b) Find the general solution of the non-homogeneous equation

$$(tD^2 - D)x = t^2, \quad t > 0$$