Mathematics 51 **Differential Equations**
Examination III **May 6, 2013, 8:30–10:30am**

Tufts UNIVERSITY

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away.**You must show all your work** in the blue book in order to receive full credit. *Please box your answers and cross out any work you do not want graded.* Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (10 points)

a. Show that $x(t) = t+4$ is a solution of the differential equation $\sin(t)D^3x+4Dx-x = -t$.

b. Write $\sin(t)D^3x + 4Dx - x = -t$ as a system of differential equations.

c. Give *one* solution to the system *in vector form.*

2. (10 points) Solve the initial-value problem $\dfrac{dx}{dt} = x^2, \quad x(0) = -3.$

3. (10 points) Find all solutions (*in vector form*) to the equation $D\vec{x} = \begin{pmatrix} 5 & 3 & 0 & 0 \\ -3 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix} \vec{x}.$

4. (10 points) Find all solutions to the equation $D\vec{x} = \begin{pmatrix} 3 & 0 \\ 0 & 5 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ e^t \end{pmatrix}$

5. (10 points) *Use the Laplace transform* to solve the initial value problem $D^2x + Dx - 2x = 2$ with $x(0) = -1$ and $x'(0) = 0$. No credit by any other method.

6. (10 points) Find all solutions to the differential equation $((D-1)^2+1)(D+3)Dx = 12.$

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7. (20 points) Consider the system $\dfrac{dx}{dt} = -x - y^2 \qquad \dfrac{dy}{dt} = y(2 - x).$

a. Is the function $E(x, y) = x^2 - y^2$ a constant of motion?

b. Is the function $E(x, y) = x^2 - y^2$ a Lyapunov function?

c. Find all equilibria of this system.

d. Find the linearization at each equilibrium and decide whether its phase portrait matches any of those at the end of the examination sheet; if so, identify which of these pictures it matches.

e. For each equilibrium decide whether the Hartman–Grobman Theorem applies.

f. Classify each equilibrium as stable or unstable.

g. Classify each equilibrium as attractor, repeller or neither.

h. Are there closed integral curves for this system?

8. (10 points) Consider the system $\dfrac{dx}{dt} = x^2 - y^2 \qquad \dfrac{dy}{dt} = yx - y.$

a. Find all equilibrium points.

b. For each equilibrium decide whether the phase portrait of the linearization matches any of those at the end of the examination sheet; if so, identify which of these pictures it matches.

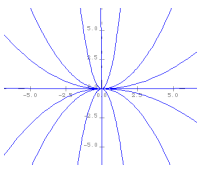
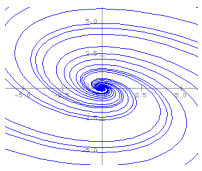
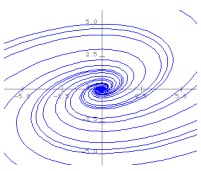
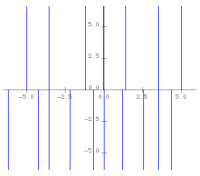
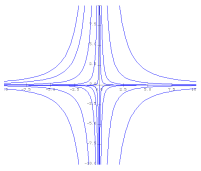
c. For each equilibrium decide whether the Hartman–Grobman Theorem applies.

9. (10 points)

a. Find a recursion formula for the coefficients in the power series (centered at 0) for the solution of $D^2x - tDx + x = 0$ with $x(0) = 0, \quad x'(0) = 1.$

b. Find the power series.

Phase portraits for matching up:

I:  **II:**  **III:**  **IV:**  **V:** 

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