No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature, you are pledging that you have neither given nor received assistance on the exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

Please start each question on a new page.

1. (10 points) Check for independence.

(a)

$$\vec{u} = \begin{pmatrix} 1 \\ 3 \\ 5 \\ 0 \\ 2 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 0 \\ 3 \\ -1 \\ 4 \\ -2 \end{pmatrix} \qquad \vec{w} = \begin{pmatrix} 3 \\ 3 \\ 17 \\ -8 \\ 10 \end{pmatrix}$$

(b)

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix} \qquad \vec{w} = \begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix}$$

2. (15 points) Solve the initial value problem $D\vec{x} = A\vec{x}$ with

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad \vec{x}(0) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

3. (15 points) Solve $D\vec{x} = A\vec{x}$ for

$$A = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -4 & 9 \\ 0 & 0 & -4 & 8 \end{pmatrix}.$$

4. (15 points) The general solution of the homogeneous system

$$D\vec{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \vec{x}$$

is

$$\vec{x} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

You do not need to verify this. Find the general solution of

$$D\vec{x} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ t \end{pmatrix}.$$

5. (10 points) Find the phase portrait of $\dfrac{dx}{dt} = (x-1)^2(x+2)$.

6. (20 points)

   (a) Write $(D^2+4)x = 0$ as a 2×2 linear system.

   (b) Find the phase portrait of the system in (a).

7. (15 points)

   (a) Verify that $E = y^2 - x^2$ is a constant of motion for the system

$$\begin{aligned} \frac{dx}{dt} &= x^2 y + y^3 \\ \frac{dy}{dt} &= x^3 + xy^2 \end{aligned} \tag{S}$$

   (b) Use (a) to find the phase portrait of (S).