Mathematics 38 — Tufts UNIVERSITY — **Differential Equations**

Exam III — **December 5, 2011**

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (10 points) Given the matrix $A = \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}$ and the eigenvector $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ find

- **a.** the eigenvalue λ of A to which $\vec{v}$ corresponds and
- **b.** the associated solution of $D\vec{x} = A\vec{x}$.

2. (20 points) The 3×3 matrix $A = \begin{pmatrix} 3 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ has a triple eigenvalue of 1 (you do not have to verify this). Find the general solution of

$$D\vec{x} = A\vec{x}.$$

3. (20 pts) Find the general solution of $\vec{x}' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \vec{x}$. Hint: $\begin{pmatrix} 1 \\ -i \end{pmatrix}$ is an eigenvector.

4. (20 pts) Find the general solution of $D\vec{x} = A\vec{x} + \begin{pmatrix} e^t \\ e^t \end{pmatrix}$, where $A = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix}$.

Hint: The general solution of $D\vec{x} = A\vec{x}$ is $c_1 e^t \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

5. (20 pts) For the system of differential equations

$$\frac{dx}{dt} = x - y$$
$$\frac{dy}{dt} = -2x + x^2$$

- **a.** find the equilibria. (Make sure to get them right; otherwise there will be little partial credit.)
  For each equilibrium
- **b.** state (with reason) whether the Hartman–Grobman Theorem applies,
- **c.** determine its stability,
- **d.** decide whether it is an attractor, a repeller, or neither.

- **e.** Sketch a plausible global phase portrait.

6. (10 pts) Check whether $E(x, y) = \sin x \sin y$ is a constant of motion for

$$\frac{dx}{dt} = -\sin x \cos y$$
$$\frac{dy}{dt} = \cos x \sin y.$$

END OF EXAMINATION