

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

- (5 points) Determine whether the spring modeled by  $(mD^2 + bD + k)x = 0$  with  $m = 1$  kg,  $k = 5$  N/m and  $b = 4$  Ns/m exhibits oscillations or not. (No credit for just saying “yes” or “no.”)
- (10 points) Find the general solution of  $x^{(4)} - 2x^{(3)} + x^{(2)} = 0$ .
- (15 points) Solve the initial-value problem  $9x'' + x = 0$  with  $x(0) = 2$  and  $x'(0) = 3$ .
- (15 points) Find the general solution of  $(D - 1)^4x = 2t + e^{-t}$ .
- (15 points) Find the general solution of

$$x'' - 2x' + x - \frac{e^t}{t^2} = 0.$$

(Check all your intermediate answers carefully; no credit for work based on wrong prior steps.)

- (15 points) Solve

$$(D - 1)x = \begin{cases} 0 & t < 2 \\ 1 & t \geq 2 \end{cases}, \quad x(0) = 1.$$

- (15 points) Find the terms up to  $t^3$  in a power series expansion for a solution of

$$x''' - 3x'' + 3x' - x = 0, \quad x(0) = 0, \quad x'(0) = 0, \quad x''(0) = 2.$$

- (10 points) Given the differential equation

$$(D^2 + 4D + 3)x = 3t + 7 \quad (\text{N})$$

- find the equivalent system  $(S_N)$ ,
- the general solution of (N) is  $x(t) = c_1e^{-t} + c_2e^{-3t} + t + 1$ . *You do not need to verify this.* Use the general solution of (N) to obtain each component of the general solution of  $(S_N)$ ,
- write  $(S_N)$  in matrix form,
- write the general solution of  $(S_N)$  in the form  $\vec{x} = c_1\vec{h}_1(t) + \cdots + c_n\vec{h}_n(t) + \vec{p}(t)$ .