Mathematics 38 **Differential Equations**
Exam II **October 31, 2011**

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (10 points) Find $\mathscr{L}[\cos^2 t]$.

2. (15 points) Solve $(D^2+1)x = \begin{cases} 0 & t < 5 \\ e^{7t} & t \geq 5 \end{cases}, \qquad x(0) = x'(0) = 0.$

3. (10 points) Consider the differential equation

(N) $$D(D+1)(D-1)x = t.$$

The corresponding homogeneous equation $D(D+1)(D-1)x = 0$ has the general solution $H(t) = c_1 + c_2 e^{-t} + c_3 e^{t}$.

a. Find the general solution of (N) by whatever method you prefer.

b. Write (N) as a 3×3 nonhomogeneous system.

4. (10 points) Make a *simplified* guess for a particular solution of

$$(D-1)^2(D^2+1)^3(D+2)x = t^2e^{3t} + e^t + e^{-t}\sin 3t + t^4.$$

5. (15 points) Find the general solution of

$$(t^2D^2 + 4tD + 2)x = t^5 \qquad (\text{for } t > 0).$$

Hint: To solve the associated homogeneous equation try solutions of the form t^α or $e^{\lambda t}$.

6. (10 points) Determine whether the spring modeled by $(mD^2 + bD + k)x = 0$ with $m = 1$ gram, $k = 100$ dynes/cm and $b = 20$ gram/s oscillates around its equilibrium.

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7. (5 points) Determine whether the system

$$\begin{aligned} x' &= -ty - z + t^2 \\ y' &= -\frac{x}{t} - \frac{z}{t} + 1 \\ z' &= x - ty \\ w' &= tx - y\sqrt{3} + z\sin t + w \end{aligned}$$

is linear. If it is linear
a. determine whether it is homogeneous, **b.** determine its order, and **c.** write it in matrix form.

8. (5 points) In parts **a.** and **b.** you are given a matrix A, a vector-valued function $\vec{E}(t)$ and formulas describing a collection of solutions of the nonhomogeneous system $D\vec{x} = A\vec{x} + \vec{E}(t)$. In each case decide whether the collection is complete.

a. $A = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}, \quad \vec{E}(t) = \begin{pmatrix} 2e^{-t} \\ -e^{-t} \end{pmatrix}: \qquad \begin{cases} x_1 = 2c_1e^{-2t} + c_2e^{-t} \\ x_2 = -c_1e^{-2t} - c_2e^{-t} + e^{-t} \end{cases}.$

b. $A = \begin{pmatrix} 5 & -3 & 0 \\ 3 & -5 & 0 \\ 0 & 1 & 2 \end{pmatrix}, \quad \vec{E}(t) = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}: \qquad \begin{cases} x_1 = 6c_1e^{4t} - 2c_2e^{-4t} \\ x_2 = 2c_1e^{4t} - 6c_2e^{-4t} \\ x_3 = c_1e^{4t} + c_2e^{-4t} - 2 \end{cases}.$

9. (10 points) Check this set of vectors for linear independence: $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 2 \\ 1 \\ 4 \\ 3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}.$

10. (5 points) Consider the functions t^3 and $|t^3|$ defined on $-\infty < t < \infty$. Are they linearly independent? Explain!

11. (5 points, *no credit unless every answer is correct*) For each of the following vectors decide whether it is an eigenvector of $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, and if so, provide the corresponding eigenvalue. For each part, your answer should be either "NO" or a number; please put all your answers on the inside front blue cover of your examination booklet.

a. $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ **b.** $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ **c.** $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ **d.** $\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$ **e.** $\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ **f.** $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ **g.** $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

h. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ **i.** $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ **j.** $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ **k.** $\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ **l.** $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ **m.** $\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$

n. $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ **o.** $\begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix}$

END OF EXAMINATION