

**Instructions:** No calculators, notes or books are allowed. Show all work to receive full credit. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

1. (15 points) Find the general solution to the differential equation

$$\frac{dx}{dt} + 2tx = 3t^2 e^{-t^2}.$$

2. (15 points) Find the general solution to the differential equation

$$(D - 2)(D - 3)x = 0$$

3. (15 points) Find the general solution to the differential equation

$$(D - 1)^3(D - 2)^5x = 0$$

4. (10 points) Apply the differential operator  $(D - 1)^3$  to the function  $e^t \ln t$ .

5. (15 points) Consider the ordinary differential equation

$$\frac{dx}{dt} = 4x^2 - x^4.$$

- (a) Find the location(s) of all equilibria.  
(b) Make a phase portrait for the equation.  
(c) Use the derivative test to determine the stability of each equilibrium.
6. (15 points) Does the Existence and Uniqueness Theorem guarantee a unique solution to the ODE

$$\frac{dx}{dt} = x^{1/2}t,$$

that satisfies  $x(0) = 0$ ? If so, find the solution. If not, determine whether there are no solutions, a unique solution (if so, find it), or more than one solution (if so, find at least two).

7. (15 points) Determine whether or not the following functions are independent on the interval  $-\infty < t < +\infty$ :

$$\begin{aligned}h_1(t) &= 1 \\h_2(t) &= e^t \\h_3(t) &= (1 + e^t)^2\end{aligned}$$

Justify your answer!