Mathematics 51 **Tufts UNIVERSITY** **Differential Equations**
Examination I **Spring 2013**

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. *Please box your answers and cross out any work you do not want graded.* Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (15 points) Consider the differential equation $x\dfrac{dx}{dt} = -t^2$.

a. Is this differential equation linear? Explain!

b. Find the solution for which $x(0) = -5$.

2. (5 points) Show that the functions t^3 and t^4 are solutions of $t^2x'' - 6tx' + 12x = 0$.

3. (5 points, no partial credit) Find all solutions of $(tD^2 - D)x = 0$ that are of the form t^α.

4. (5 points, no partial credit) Evaluate $\det\begin{pmatrix} 0 & 1 & 6 & 9 & 1 \\ 0 & 0 & 2 & 7 & 0 \\ 0 & 0 & 0 & 3 & 8 \\ 0 & 0 & 0 & 0 & 4 \\ 5 & 0 & 0 & 0 & 0 \end{pmatrix}$.

5. (10 points) A savings account pays 3% interest per year, compounded continuously. In addition, the income from another investment is credited to the account continuously, at the rate of $700 per year. Set up a differential equation to model this account.

6. (5 points, no partial credit) Find the general solution of $(D-1)^2(D+1)x = 0$.

7. (7 points, no partial credit) Find the general solution of $3(D^2 + D + 2)^2x = 0$.

8. (8 points, no partial credit) Make a *simplified* guess for a particular solution of

$$(D-1)(D^2+1)^3(D+2)x = t^2e^t + e^{-t}\sin 3t + t.$$

Do not evaluate the constants.

9. (15 points) Find (and simplify where possible) the general solution of $x'' - 2x' + x = e^t/t^2$. (*Check all your intermediate answers carefully; no credit for work based on wrong prior steps.*)

10. (10 points)

a. Compute the Wronskian of $h_1(t) = te^t$ and $h_2(t) = t^2e^t$ at $t = 1$.

b. Are these 2 functions linearly independent?

11. (10 points) Are the functions t^5, $|t|^5$ linearly independent on $(-\infty, \infty)$? *Justify your conclusion.*

12. (5 points) Suppose $f(t)$ is continuous. Solve the initial-value problem $x' + f(t)x = 0, \quad x(1) = 0$. (Hint: Think before applying standard techniques.)

END OF EXAMINATION