

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature, you are pledging that you have neither given nor received assistance on the exam. Any violations will be reported to the appropriate dean, and will result in an F for the course. *Please start each problem on a new page.*

1. (10 points) Compute

$$\det \begin{bmatrix} 0 & -1 & 3 & 2 \\ 5 & 0 & 2 & 1 \\ 6 & 0 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$$

2. (20 points) Consider the first order o.d.e

$$(A) \quad (t-1) \frac{dx}{dt} = -x$$

- (a) Find the largest rectangular region in the tx -plane that contains $(0,0)$ on which the hypotheses of the existence and uniqueness theorem (E & UT) hold.
(b) Do the hypotheses of the E & UT hold for the point $(1,0)$?
(c) Do you have a solution of (A) with $x(1) = 0$?
(d) Is it unique? Justify.

3. (10 points) Show that $h_1(t) = e^t$ and $h_2(t) = e^{2t}$ are linearly independent.

4. (14 points) Show that the functions $|t|$, t and t^2 for $-\infty < t < \infty$ are linearly independent.

5. (28 points)

- (a) Check that $h_1(t) = \frac{1}{t}$ and $h_2(t) = \frac{2}{t^2}$ are solutions of

$$(H) \quad (t^2 D^2 + 4tD + 2)x = 0 \quad t > 0$$

- (b) Find the general solution. Explain why this is the general solution.
(c) Find a constant solution for

$$(N) \quad (t^2 D^2 + 4tD + 2)x = 1$$

- (d) Find a solution for (N) that satisfies $x(1) = 0$, $x'(1) = 0$. Is it unique? Justify.

6. (18 points) Solve the system

$$\begin{aligned} x_1 + 2x_2 + x_3 - x_4 - x_5 &= 2 \\ 2x_1 + 2x_2 + 2x_3 - 3x_4 - 2x_5 &= 1 \\ -x_1 - x_3 + 2x_4 + x_5 &= 1 \end{aligned}$$