Tufts University

Exam 1 | Department of Mathematics | February 14, 2011

Mathematics 38

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. You must show all your work in the blue book in order to receive full credit. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam.

1. (14 points) Consider the equation

$$tx' + (1-t)x^2 = 0$$

Please, answer the following questions (NO PARTIAL CREDIT):

(a) Is the given differential equation separable?

(b) Is it linear?

(c) Give the largest interval containing $t = 1$ where the equation is normal.

(d) Write the equation in the standard form.

(e) Find the general solution.

(f) Find a solution satisfying the initial condition $x(1) = -2$.

(g) Why is the solution satisfying the initial condition $x(1) = -2$ unique? (Please, refer to an appropriate theorem).

2. (10 points) Consider the differential equation

$$(t-1)\frac{dx}{dt} = x.$$

(a) Determine the largest rectangular region of the (t, x)-plane that contains the point $(0, 2)$ and on which the hypotheses of the existence and uniqueness theorem hold for the given o.d.e.

(b) Determine whether there is no solution, a unique solution, or more than one solutions passing through the point $(1, 2)$.

3 (10 points) Solve the following first order linear differential equation

$$2x' - x = te^t.$$

Exam continues on other side

4 (8 points) (NO PARTIAL CREDIT)

(a) Determine whether the system of linear algebraic equations

$$x - 2y + z = 0$$

$$3x + 2y + z = 0$$

$$y - z = 0$$

has a unique solution, infinitely many solutions, or no solution;

(b) If the system has a unique solution, then find this solution.

5. (6 points) Use the Wronskian test for independence to show that functions e^{at} and e^{bt}, with $a \neq b$, are linearly independent.

6. (10 points) Use the exponential shift formula to compute the following expressions:

   (a) $(D^2 + D - 5)[te^{2t}]$

   (b) $(D - 5)^4[e^{5t}\sin(t)]$

7. (15 points) Consider the following nonhomogeneous second order differential equation

$$x'' - 2x' + x = t^2.$$

   (a) Find the general solution of the corresponding homogeneous equation;

   (b) Find a particular solution of the form $x(t) = At^2 + Bt + C$ to the nonhomogeneous equation;

   (c) Find the solution of the given equation satisfying the conditions

$$x(0) = x'(0) = 0.$$

8. (7 points) Find the annihilator of smallest possible order for the function (NO PARTIAL CREDIT)

$$t^2 + e^t \cos 2t - 1$$

9. (20 points) Solve the following initial-value problem

$$(D^2 - 1)(D^2 + 1)x = 0$$

$$x(0) = 1,\ x'(0) = x''(0) = x'''(0) = 0.$$