Math 42	Tufts University	Monday, May 9, 2016
Calculus III	Department of Mathematics	8:30–10:30am
	Final Exam	

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the space provided in order to receive full credit. A correct answer with no work may not necessarily score any points. Please box your answers and cross out any work you do not want graded. Make sure to sign your exam in the space provided. With your signature, you are pledging that you have neither given nor received assistance on the exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

Question	Points	Score
1	4	
2	4	
3	4	
4	4	
5	12	
6	10	
7	10	
8	12	
9	10	
10	10	
11	10	
12	10	
Total:	100	

No partial credit for questions 1-4

1. (4 points) Let $\phi(x, y)$ be a scalar function with continuous partial derivatives on the plane. Let $\vec{F} = \nabla\phi$ and let C be the curve given by $\phi(x, y) = 5$. Assume further that C is closed. Is the circulation of $\vec{F}$ along C positive, negative, or zero?

2. (4 points) Let $\vec{F}$ be a continuous vector field on the plane. Let C_1, C_2 be two smooth curves on the plane with the same starting and ending points. Suppose $\int\limits_{C_1} \vec{F} \cdot d\vec{r} = \int\limits_{C_2} \vec{F} \cdot d\vec{r}$. Does the fundamental theorem of line integrals imply that $\vec{F}$ is a conservative field?

3. (4 points) True or false: $\vec{r}(t) =< t, t^2, t^3 >$ is parameterized by arc length.

4. (4 points) Let $\vec{F}$ be a continuous and differentiable vector field. Is the expression $\nabla \cdot (\nabla \cdot \vec{F})$ defined?

There is partial credit for questions 5-12

5. (12 points) Let $f(x, y) = \ln(1 + x + y)$.

   (a) (6 points) Find an equation for the tangent plane to $z = f(x, y)$ at the point $(0, 0)$.

   (b) (6 points) Using linear approximation, estimate the value of $f(0.1, 0.2)$.

6. (10 points) Let R be the region bounded by $x = y^2$ and $x = 18 - y^2$. Use a line integral to find the area of R (no credit will be given for a method that does not use a line integral).

7. (10 points) Find the absolute maximum and absolute minimum of

$$f(x, y) = xy + y - x^3$$

on the region $R = \{(x, y) : x^2 \leq y \leq 9\}$

 Exam continues...

8. (12 points) Let $\vec{F}(x, y, z) =< yz, xz + z, xy + y >$.

(a) (2 points) Show that $\vec{F}$ is a conservative vector field.

(b) (6 points) Find a potential function ϕ for $\vec{F}$.

Exam continues...

(c) (2 points) Use the potential function you found in (b) to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the line segment from $(3, 3, 3)$ to $(2, 2, 2)$.

(d) (2 points) Briefly justify why you can use a potential function to evaluate this integral - i.e., name or state a theorem that lets you do this.

 Exam continues...

9. (10 points) Find the volume of the solid bounded by $x^2 + y^2 = 9, \quad z = 3, z = 0$, and $y = z$ using cylindrical coordinates.

10. (10 points) Use Green's Theorem to evaluate

$$\int_C (-3y - e^{x^2})dx + (x - e^{y^2})dy$$

where C is the boundary of the half-disk $\{(x, y) : x^2 + y^2 \leq 2, y \geq 0\}$
(No credit will be given for a solution that does not use Green's Theorem).

11. (10 points) Use the Divergence Theorem to compute the outward flux of $\vec{F} = < -x, x - y, x - z >$ across S, where S is the surface of a cube cut from the first octant by the planes $x = 2$, $y = 2$, and $z = 2$ (Indicate where you use the Divergence Theorem. No credit will be given for a solution that does not use the Divergence Theorem).

12. (10 points) Use Stokes' Theorem to compute

$$\iint_S (\nabla \times \vec{F}) \cdot \vec{n} \; ds$$

where $\vec{F} =< -z, x, y >$ and S is the hyperboloid

$$z = 10 - \sqrt{1 + x^2 + y^2} \quad z \geq 0$$

(Indicate where you use Stokes' Theorem. No credit will be given for a solution that does not use Stokes' Theorem).

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Exam continues. . .

I pledge that I have neither given nor received assistance on this exam.

Name: ______________________________

Signature: ______________________________

Date: ______________________________

Section Number: ______________________________

Instructor's Name: ______________________________