Math 42 | Tufts University | Monday, April 4, 2016
Calculus III | Department of Mathematics | 12:00–1:20pm
Exam 2

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the space provided in order to receive full credit. A correct answer with no work may not necessarily score any points. Please box your answers and cross out any work you do not want graded. Make sure to sign your exam in the space provided. With your signature, you are pledging that you have neither given nor received assistance on the exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

I pledge that I have neither given nor received assistance on this exam.

Name: _____

Signature: _____

Date: _____

1. (10 points) Consider the surface $x^2 + y^2 + z^2 - 3 = 0$ which may be regarded as a level curve of $w = x^2 + y^2 + z^2 - 3$. A point of that surface is $(1, 1, 1)$.

   (a) Find the gradient of w and evaluate it at $(1, 1, 1)$.

   (b) Find an equation of the tangent plane to $w = 0$ at $(1, 1, 1)$.

Exam continues...

2. (15 points) Use differentials to approximate the change in $z = e^{x+y}$ when (x, y) changes from $(0, 0)$ to $(0.1, -0.05)$

3. (15 points) Find the critical points of $f(x, y) = ye^x - e^y$ and use the second derivative test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point.

Exam continues...

4. (15 points) Find the point(s) on the cone

$$z^2 = x^2 + y^2$$

closest to $(0, -1, 0)$.
Hint: look at the distance squared.

5. (10 points) Evaluate

$$\int_0^{\pi} \int_x^{\pi} \sin y^2 \, dy \, dx$$

Exam continues...

6. (10 points) Evaluate $\iint_R 2xy$ where $R = \{(x, y) : x^2 + y^2 \leq 4, x \geq 0\}$

Exam continues...

7. (14 points)

(a) (2 points) Write $z = \sqrt{4 - x^2 - y^2}$ in spherical coordinates.

(b) (4 points) Write $z = \dfrac{1}{\sqrt{3}}\sqrt{x^2 + y^2} = \dfrac{\sqrt{3}}{3}\sqrt{x^2 + y^2}$ in spherical coordinates.

Exam continues...

(c) (4 points) Using spherical coordinates, write a triple integral for the volume of the solid bounded by $z = \sqrt{4 - x^2 - y^2}$ and $z = \dfrac{1}{\sqrt{3}}\sqrt{x^2 + y^2}$

(d) (4 points) Evaluate the integral from part (c).

8. (11 points) Consider the function $\vec{F} =< -y, x^2 >$. Let C be line segment from $(1, 0)$ to $(0, 1)$.

(a) (2 points) Sketch the vector field $\vec{F}$ in the domain $[0, 1] \times [0, 1] \subseteq \mathbb{R}^2$.

(b) (1 points) Sketch C and give a parametrization of $\vec{r}(t)$.

(c) (2 points) Compute $\int_C |\vec{F}|^2 ds$

Exam continues...

(d) (3 points) Compute $\int_C \vec{F} \cdot \vec{T} ds$

(e) (3 points) Compute $\int_C \vec{F} \cdot \vec{n} ds$

Problem	Point Value	Points
1	10	
2	15	
3	15	
4	15	
5	10	
6	10	
7	14	
8	11	
	100	