Math 42 | Tufts University | Monday, February 22, 2016
Calculus III | Department of Mathematics | 12:00–1:20pm
Exam 1

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. Please box your answers and cross out any work you do not want graded. Make sure to sign your blue book. With your signature, you are pledging that you have neither given nor received assistance on the exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

1. (4 points) **No partial credit:** True or false: if $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$ and $|\mathbf{u} \times \mathbf{v}| = \mathbf{u} \cdot \mathbf{v}$**=0** then $\mathbf{u}$**=0** or $\mathbf{v}$**=0**.

2. (4 points) **No partial credit:** True or false: the area of the triangle ABC is $|\overrightarrow{AB} \times \overrightarrow{AC}|$.

3. (4 points) **No partial credit**: True or false: $proj_{\mathbf{u}}(\mathbf{v})$ is parallel to $\mathbf{u}$.

   All other questions have partial credit.

4. (6 points) Let $\begin{array}{l}\mathbf{u} =< 2, -1, 2 > \\ \mathbf{v} =< 3, 4, 0 >\end{array}$ Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$. Find $\cos\theta$. Simplify your answer.

5. (12 points) Let $\mathbf{r}(t) =< 2t + 4, 6t >$ be the position of particle 1 at time t and let $\mathbf{m}(s) =< 2s, s + 3 >$ be the position of particle 2 at time s.
   (a) (8 points) Do the lines traced by $\mathbf{r}(t)$ and $\mathbf{m}(s)$ intersect? If so, where?
   (b) (4 points) Do the particles collide? Why or why not? Assume that $t \geq 0$ and $s \geq 0$ measure time in seconds, and that motion starts at $s = t = 0$.

6. (15 points) Let $f(x, y) = \dfrac{x^2 + y^2 - 1}{x^2 + y^2 + 1}$.
   (a) Find the domain of $f(x, y)$.
   (b) Find the level curves of $f(x, y)$ and identify the shape they have.
   (c) Sketch the level curves of $f(x, y)$ for $k = 0, \frac{1}{2}$, and $\frac{1}{3}$.

7. (15 points) Find the distance from $P : (1, 3, 8)$ to the plane $x - 2y - z = 12$. You must use the fact that $Q : (1, -6, 1)$ is on the plane. Simplify your answer.
   Hint: Use an orthogonal projection.

8. (15 points) Let $f(x, y) = e^y \ln(x + y)$
   (a) Find the domain of $f(x, y)$.
   (b) Find all first order partial derivatives.
   (c) Find all the second order partial derivatives.

9. (15 points) Let Q be the plane $-x + 2y + z = 1$ and R the plane $x + y + z = 0$. Find the intersection of the two planes and parameterize it. You must use the fact that a point in the intersection is $(0, 1, -1)$.

10. (10 points) Find the arc length of the curve given by $\mathbf{r}(t) =< 2t^{\frac{9}{2}}, t^3 >$ for $0 \leq t \leq 1$.