Tufts University

Math 42 | **Department of Mathematics** | **May 4, 2015**

Final Exam | **8:30 am to 10:30 am**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) Write parametric equations for the line of intersection of the two planes $x - 4z = 0$ and $x - y + z = 5$.

2. (15 points) Let $f(x, y, z) = ze^{(2x+2y)}$.

   (a) Find a unit vector in the direction in which f increases the fastest at the point $(0, 0, 1)$ and give the largest rate of change of the function at this point.

   (b) The equation $ze^{(2x+2y)} = 1$ defines z as an implict function of x, y near the point $(0, 0, 1)$. Using implicit differentiation, find $\frac{\partial z}{\partial x}$ at the point $(0, 0, 1)$.

3. (10 points) A surface is given parametrically by $\mathbf{r}(u, v) =< u\cos v, u^2, u\sin v >$ where $u \geq 0, 0 \leq v \leq 2\pi$.

   (a) Write an equation in x, y, z, $F(x, y, z) = 0$ for the surface.

   (b) Give a rough sketch of the surface.

4. (10 points) Consider the following integral $\int_0^6 \int_{-\sqrt{6x-x^2}}^{\sqrt{6x-x^2}} y \, dydx$

   (a) Sketch the region of integration.

   (b) Write (but **DO NOT EVALUATE**) the integral as an equivalent integral in polar coordinates .

5. (10 points) Let $f(x, y) = x^2 + y^2 - 2x$. Find the absolute maximum and the absolute minimum of f on the disk (interior and boundary) given by $x^2 + y^2 \leq 4$.

6. (15 points) Consider the vector field $\mathbf{F} =< y + 3,\ x + \sin(y^2) >$.

   (a) Let $\mathcal{C}$ be the straight line segment from $(-2, 0)$ to $(2, 0)$. Parameterize this segment and compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$

   (b) Show that $\mathbf{F}$ is conservative. We **DO NOT** recommend that you try to find a potential function.

   (c) Let $\mathcal{C}_1$ be the lower half of the circle of radius two centered at the origin $x^2 + y^2 = 4, y \leq 0$ oriented from left to right. Compute $\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r}$

7. (10 points) Let $\mathcal{R}$ be the region contained in the two spheres $x^2 + y^2 + z^2 \leq 4$ and $x^2 + y^2 + (z-2)^2 \leq 4$. Use spherical coordinates to compute the volume of $\mathcal{R}$.

8. (10 points) Consider the vector field $\mathbf{F} =< y + e^{x^2},\ x + \ln(y^2 + 1), x >$. Let $\mathcal{C}$ be the triangle obtained as the intersection of the plane $x + y + z = 8$ with the first octant oriented counterclockwise when viewed from above. Compute $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

9. (10 points) Consider the vector field $\mathbf{F} =< x^3 + e^{yz}, y^3 + e^{xz}, 6z >$. Let $\mathcal{S}$ be the boundary of the cylinder $x^2 + y^2 \leq 4, 0 \leq z \leq 3$ (side of the cylinder plus top and bottom disks). Compute the outward flux of $\mathbf{F}$ across the boundary of $\mathcal{S}$.