Tufts University

Math 42 | Department of Mathematics | April 1, 2013

Exam 2 | Noon–1:20 p.m.

**Instructions:** Please do all nine problems below. Except for Problem 1, you must show your work and justify your answers in order for partial or full credit to be awarded. No books, notes or calculators are allowed during this exam. *You are required to sign each examination blue book that you are handing in. With your signature, you are pledging that you have neither given nor received any help pertaining to this exam. If you are found in violation of this policy, you will be refered to the Dean of Students and automatically receive an* **F** *for the course.*

*You may assume that the volume enclosed by a sphere of radius $r$ is* $\frac{4}{3}\pi r^3$.

1. (10 points) **True or False – no partial credit.** On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) If $D$ is the disk in the plane given by $x^2 + y^2 \leq r^2$, then

   $$\iint_D \sqrt{r^2 - x^2 - y^2}\, dx\, dy = \frac{1}{2} \cdot \frac{4}{3}\pi r^3.$$

   (b) $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx = \int_0^{2\pi} \int_0^1 e^{r^2}\, dr\, d\theta.$

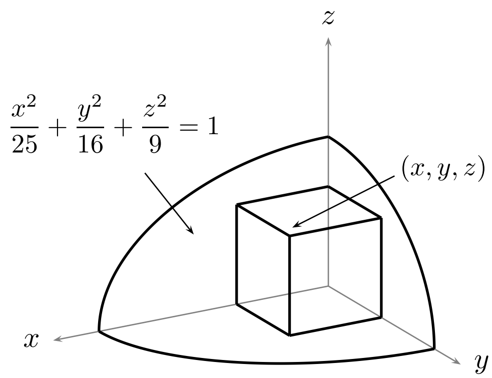
   (c) If the point $P$ has spherical coordinates $(\rho, \varphi, \theta) = (4, \pi/4, \pi/3)$, then its Cartesian coordinates are $(x, y, z) = (\sqrt{2}, \sqrt{6}, 2\sqrt{2})$.

   (d) If $S$ is a surface whose equation in spherical coordinates is $\rho \cos\varphi = 3$, then $S$ is a plane.

   (e) $\displaystyle\int_0^2 \int_0^x f(x, y)\, dy\, dx = \int_0^2 \int_0^y f(x, y)\, dx\, dy.$

**The exam continues on the opposite side of this sheet.**

2. (12 points) A box with faces parallel to the coordinate planes lies in the first octant inside the ellipsoid $\dfrac{x^2}{25} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = 1$. (See the figure below.) The volume of the largest such box is to be found using Lagrange multipliers.



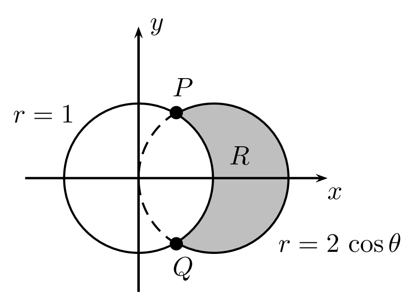

   (a) Write down the system of four equations in $x$, $y$, $z$, and $\lambda$ which you will need to solve for the vertex $(x, y, z)$ of the largest box.
   (b) Solve the system you obtained in Part (a) for $x$, $y$, and $z$.

3. (10 points) Consider the double integral:

$$\int_0^2 \int_{x^2}^4 x\, e^{y^2}\, dy\, dx.$$

   (a) Sketch the region of integration and label the boundary curves.
   (b) Switch the order of integration and evaluate the double integral.

4. (10 points) Let $R$ be the region in the plane outside the unit circle $r = 1$ and inside the circle $r = 2\cos\theta$. (See the figure below.)



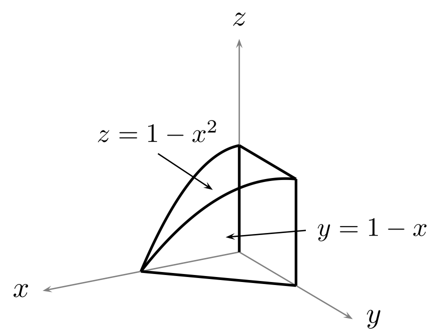

   (a) The two circles intersect at the points $P$ and $Q$. Find the polar coordinates of $P$ and $Q$.
   (b) Express the double integral

$$\iint_R \frac{1}{1 + x^2 + y^2}\, dx\, dy$$

   as an iterated double integral in polar coordinates. **DO NOT EVALUATE.**

**The exam continues on the next sheet.**

5. (12 points) Let $E$ be the solid in the first octant bounded by the parabolic cylinder $z = 1 - x^2$ and by the plane $y = 1 - x$. (See the figure below.)




Express the triple integral $\iiint_E f(x, y, z)\, dV$ as an iterated triple integral

(a) in the order $dz\, dy\, dx$

(b) in the order $dy\, dx\, dz$.

6. (12 points) Express the following iterated triple integral

$$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}\, dz\, dy\, dx$$

in spherical coordinates. **DO NOT EVALUATE.**

7. (12 points) Let $E$ be the solid outside the cylinder $x^2 + y^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

(a) Express the volume of $E$ as an iterated triple integral in cylindrical coordinates.

(b) Evaluate the integral you obtained in Part (a).

8. (12 points) Evaluate the line integral

$$\int_C x\, ds,$$

where $C$ is the arc of the helix $x = 3\cos t$, $y = 3\sin t$, $z = 4t$ from $(3, 0, 0)$ to $(0, 3, 2\pi)$.

9. (10 points) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{F} = (y + z)\,\mathbf{i} - 2x\,\mathbf{j} + 3z\,\mathbf{k}$, and $C$ is the curve parametrized by $\mathbf{r}(t) = \langle t^2, t, t^3 \rangle$ for $0 \le t \le 1$.

**End of Exam.**