Tufts University

Math 42 | Department of Mathematics | February 21, 2013

Exam 1 | Noon–1:20 p.m.

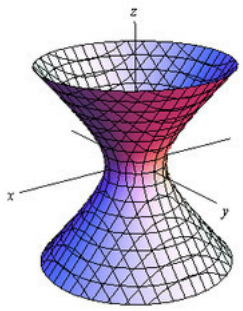
Instructions: Please do all nine problems below. Except for Problem 1, you must show your work and justify your answers in order for partial or full credit to be awarded. No books, notes or calculators are allowed during this exam. *You are required to sign each examination blue book that you are handing in. With your signature, you are pledging that you have neither given nor received any help pertaining to this exam. If you are found in violation of this policy, you will be refered to the Dean of Students and automatically receive an* **F** *for the course.*

1. (10 points) **True or False – no partial credit.** On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) The cross product of two unit vectors is a unit vector.

   (b) The line $x = 3 - 2t,\ y = 4 + 4t,\ z = -1 - 2t$ is perpendicular to the plane $x - 2y + z = 7$.

   (c) The surface below is the graph of the equation $x^2 + y^2 = 3z^2 - 1$.



   (d) The curve $\mathbf{r}(t) = 3\sin t\,\mathbf{i} - 3\cos t\,\mathbf{j} + 4t\,\mathbf{k}$ uses arc length as a parameter.

   (e) If $\nabla f(a, b) = \mathbf{0}$, then the tangent plane to the graph of $z = f(x, y)$ above the point (a, b) is parallel to the (x, y)-plane.

2. (12 points) Consider the triangle with vertices $P(3, 1, 0),\ Q(4, 3, 1)$, and $R(5, 2, 0)$.

   (a) Find the area of the triangle ΔPQR.

   (b) Find an equation of the plane containing the triangle ΔPQR.

3. (6 points) Let $\mathbf{u} = 5\,\mathbf{i} + 3\,\mathbf{j} + 8\,\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$.

   (a) Find the scalar projection $\text{scal}_{\mathbf{v}}\mathbf{u}$.

   (b) Find the scalar projection $\text{proj}_{\mathbf{v}}\mathbf{u}$.

The exam continues on the opposite side of this sheet.

4. (15 points) A disoriented mosquito is flying along a path such that its position at time t is $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 4\cos t\,\mathbf{j} + 5\sin t\,\mathbf{k}$.

   (a) Determine the mosquito's velocity and acceleration when $t = \frac{\pi}{2}$.

   (b) Find parametric equations for the tangent line to the curve above when $t = \frac{\pi}{2}$.

   (c) Calculate the distance the mosquito has traveled from $t = 0$ to $t = 2$.

5. (10 points) Find parametric equations of the line of intersection of the two planes $x+y+z = 3$ and $2x - y + 3z = 4$. (You may use the fact that the point $P(1, 1, 1)$ lies in both planes.)

6. (8 points) Suppose that $z = \dfrac{x}{y}$, and $x = s\,e^t$, $y = s + t$. Find $\dfrac{\partial z}{\partial t}$ when $s = 2$ and $t = 0$.

7. (15 points) Let $f(x, y, z) = x^2 - 2y^2 + z^2$.

   (a) Find a unit vector in the direction of which $f(x, y, z)$ is increasing the fastest at the point $(1, -1, 2)$.

   (b) Calculate the directional derivative of $f(x, y, z)$ in the direction of the unit vector $\mathbf{v} = \dfrac{2}{3}\,\mathbf{i} + \dfrac{2}{3}\,\mathbf{j} + \dfrac{1}{3}\mathbf{k}$ at the point $(1, -2, 2)$.

   (c) Find an equation of the tangent plane to the level surface $f(x, y, z) = 3$ at the point $(1, -1, 2)$.

8. (12 points) Let $f(x, y) = x^2 - xy + 3y^2$.

   (a) Find the linear approximation $L(x, y)$ of $f(x, y)$ at the point $(3, -1)$.

   (b) Use $L(x, y)$ to estimate $f(2.96, -0.95)$. Give your answer as a decimal expressed to two places.

9. (12 points)

   (a) Find all critical points of the function $f(x, y) = xy(1 - x - y)$.

   (b) The origin $(0, 0)$ is a critical point of the function $f(x, y) = x^2y^3 - x^2 + 2y^2$. Determine whether it corresponds to a local maximum, a local minimum, or a saddle point.

End of Exam.