1. (10 points) **True or False - No Partial Credit:** On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) Let \( u, v, \) and \( w \) be three nonzero vectors in \( \mathbb{R}^3 \). Then, \( u \cdot (v \cdot w) = (u \cdot v) \cdot w \).

   (b) Let \( g(x, y) \) be a continuous function defined on the domain \( D = \{(x, y) \mid a \leq x \leq b, c \leq y \leq d\} \).
   The integral
   \[
   \iint_D g(x, y) dA = \left( \int_a^b g(x, y) \, dx \right) \left( \int_c^d g(x, y) \, dy \right).
   \]

   (c) Let \( F \) be a vector field with continuous partial derivatives. Then, \( \nabla \cdot (\nabla \times F) = 0 \).

   (d) Let \( F = \langle -\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}, z \rangle \). Stokes’ Theorem says that \( \iint \nabla \times F \cdot dS = \int_C F \cdot dr \) for \( C \) being the closed boundary curve of the top half of the unit sphere, \( S \).

   (e) Let \( \nabla \cdot F = 1 \) and let \( S \) be the surface of the unit cube, \( 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1 \), with outward unit normal. Then, \( \int_S F \cdot dS = 1 \).

2. (10 points) Let \( r(t) = (2 + t) \mathbf{i} + (t^2 + 3) \mathbf{j} + \left( \frac{2}{3}t^3 - 5 \right) \mathbf{k} \).

   (a) Find the parametric equations for the tangent line to the curve \( r(t) \) at \( t = 3 \).

   (b) Find the arc length of \( r(t) \) from \( t = 0 \) to \( t = 3 \).

3. (10 points) Find the absolute minimum and maximum for the function \( f(x, y) = 2x + 2y^2 + 1 \) on the unit disk \( D = \{(x, y) \mid x^2 + y^2 \leq 1\} \).

4. (10 points) Let \( f(x, y, z) = x^2 + y^2 + z^2 \), and consider the integral
   \[
   I = \int_0^2 \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-y^2-z^2}}^{\sqrt{4-y^2-z^2}} f(x, y, z) \, dx \, dz \, dy.
   \]

   (a) Rewrite the integral in spherical coordinates. **Do not evaluate this integral (yet).**

   (b) Evaluate either the original integral or the one in part (a).

The exam continues on the back!
5. (10 points) Let $\mathbf{F}(x, y) = \langle 2x, -y \rangle$.

(a) Is $\mathbf{F}$ a conservative field? If so, find a potential function for $\mathbf{F}$.

(b) Let $\mathbf{C}$ be the curve parameterized by $\mathbf{r}(t) = \langle \sin(t), t \rangle$, for $0 \leq t \leq \frac{\pi}{2}$. Evaluate $\int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{r}$.

6. (15 points) Using Green’s Theorem, compute the area of the region, $D$, between the $x$-axis and the tent-shaped curve parametrized as follows:

$$\mathbf{r}(t) = \langle \cos^3(t), \sin^2(t) \rangle \quad \text{for} \quad 0 \leq t \leq \pi.$$ 

Make sure to explicitly show how you are using Green’s Theorem to get full credit. A figure of the region $D$ is shown below:

![Diagram of the region D]

Will Jumbo fit in the tent?
(Not really an exam question...)

7. (10 points) Let $S$ be the surface parametrized by

$$\mathbf{t}(r, \theta) = \langle r \cos(\theta), r \sin(\theta), r \rangle, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq r \leq 2.$$ 

(a) Sketch the surface.

(b) Evaluate $\iint_{S} z \, dS$

8. (15 points) Let $\mathbf{F} = \langle y^2, z^2, x^2 \rangle$ and let $\mathbf{C}$ be the curve that bounds the triangular plate, $x + y + z = 1$, in the first octant ($x > 0, y > 0, z > 0$), oriented clockwise when viewed from above.

Use Stokes’ Theorem to compute the circulation, $\oint_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{r}$.

9. (10 points) Let $\mathbf{F}(x, y, z) = \langle x^2, y, x - z \rangle$ and $S$ be the boundary surfaces of the region contained in the cylinder $x^2 + y^2 = 1$ between the planes $z = y$ and $z = 2$, with outward unit normal.

Evaluate $\iint_{S} \mathbf{F} \cdot d\mathbf{S}$.

End of Exam

According to a recent survey, 100% of all people say they participate in surveys.