Tufts University

Math 13 — **Department of Mathematics** — **April 2, 2012**

Exam 2 — **12:00 pm to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) **True or False - No Partial Credit**: On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) If $D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$, then the area of D is given by $\displaystyle\int_{g_1(x)}^{g_2(x)} \int_a^b 1\, dx\, dy$.

   (b) The integral $\int_0^{2\pi} \int_0^2 \int_r^2 dz\, dr\, d\theta$ represents the volume enclosed by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 2$.

   (c) The sphere, $x^2+y^2+z^2 = 2x$, in Cartesian coordinates is represented as $\rho = 2\cos(\theta)\sin(\phi)$ in Spherical coordinates.

   (d) Let $\mathbf{F}(x,y)$ be conservative. If C is the unit circle, then $\int_C \mathbf{F}(x,y) \cdot d\mathbf{r} = 0$.

   (e) The vector field $\mathbf{F}(x,y,z) = \langle x, y, z\rangle$ is conservative on $\mathbb{R}^3$.

2. (10 points) Evaluate the following integrals. You may use any transformations or integration rules that you wish, but you must explain all of your work.

   (a) $\iint_R 2x\cos(x^2)\, dA,\ R = \{(x,y) \mid 0 \le x \le \sqrt{\pi},\ 0 \le y \le \pi\}$.

   (b) $\int_0^{\pi} \int_y^{\pi} \cos(x^2)\, dx\, dy$.

3. (10 points) Consider the integral $\displaystyle\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 2xy^2\, dy\, dx$.

   (a) Sketch the region of integration.

   (b) Rewrite the integral in polar coordinates.

   (c) Evaluate the integral, in **either** Cartesian or polar coordinates.

4. (10 points) Let V be the solid region bounded by $4z = x^2 + y^2$ and $z = 4$. Express $\iiint_V f(x,y,z)\, dV$ in the following two orders: $dz\, dy\, dx$ and $dy\, dx\, dz$.
   Do not evaluate the integrals.

The exam continues on the back!

5. (10 points) Compute $\iiint\limits_V \frac{1}{x^2+y^2+z^2}\,dV$, where V is the solid region between two concentric spheres centered at the origin, $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.

6. (10 points) Let $\mathbf{F}(x,y,z) = \langle y, 5z, 4x\rangle$. Compute $\int\limits_C \mathbf{F}(x,y,z)\cdot d\mathbf{r}$, where C is defined by $\mathbf{r}(t) = \langle t, t^2, t^3\rangle \quad 0 \le t \le 1$.

7. (15 points) Consider $f(x,y,z) = x+y+z$.
   (a) Find the line integral of $f(x,y,z)$ over the line segment from the point $(1,2,3)$ to $(-1,4,4)$.
   (b) What is the line integral of $\nabla f(x,y,z)$ over the same line segment?

8. (15 points) Let $\mathbf{F}(x,y,z) = \langle 2xy+z, x^2+\cos(y), x\rangle$. Find the potential function of $\mathbf{F}(x,y,z)$ if it exists, or show that one does not exist.

9. (10 points) Consider the solid E bounded by the sphere $x^2+y^2+z^2=2z$ and the sphere $x^2+y^2+z^2=1$. Set up and evaluate a triple integral in *Cylindrical Coordinates* to calculate the volume of E.

End of Exam

There are 10 types of people in the world.
Those who know binary and those who don't.