Tufts University

Math 13	**Department of Mathematics**	**Februrary 27, 2012**
	Exam 1	**12:00 pm to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) **True or False - No Partial Credit**: On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$. The magnitude of $\mathbf{u}+\mathbf{v}$ is at least the magnitude of $\mathbf{u}$.

   (b) The vectors $\mathbf{u} = \langle 3, 2, 1\rangle$ and $\mathbf{v} = \langle -6, -4, -2\rangle$ are parallel.

   (c) The vector $\mathbf{w} = \mathbf{i} - \mathbf{j}$ is orthogonal to the vector $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

   (d) The directional derivative of $f(x, y) = xy$ in the direction of the vector $\mathbf{u} = \langle 1, 1\rangle$ at the point $(1, 1)$ is 2.

   (e) $\mathbf{u} - \mathbf{proj_v u}$ is orthogonal to $\mathbf{v}$.

2. (10 points) Consider the triangle that has vertices $(1, 1, 1)$, $(2, 2, 1)$, and $(1, 2, 2)$.

   (a) What is the area of the triangle?

   (b) Write an equation of the plane that contains these three points.

3. (10 points) A roller coaster travels on a track with position given by

$$\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + \sin(2t)\mathbf{k} \text{ for } 0 \le t \le 2\pi.$$

   (a) Plot $\mathbf{r}(t)$ for $0 \le t \le 2\pi$.

   (b) Find the velocity and acceleration of the roller coaster as functions of t.

   (c) Find the unit tangent vector to $\mathbf{r}(t)$ at $t = \frac{\pi}{2}$.

4. (15 points) Consider the surface $z = f(x, y) = -4x^2 - y^2$.

   (a) Sketch several level curves of $f(x, y)$, clearly labeling the curves.

   (b) Sketch several xz-traces and yz-traces of $z = -4x^2 - y^2$, clearly labeling the traces.

   (c) Give the name for the surface $z = -4x^2 - y^2$.

The exam continues on the back!

5. (10 points) Let $f(x, y, z)$ be a function in three variables, which satisfies

$$f_x(4,1,2) = 3, \quad f_y(4,1,2) = 7, \quad f_z(4,1,2) = 5.$$

Let

$$x(s,t) = s + 2t, \quad y(s,t) = t^2, \quad z(s,t) = st.$$

Find $\frac{\partial f}{\partial t}$ at the point $(s,t) = (2,1)$.

6. (10 points) A spaceship is in trouble near the sunny side of Mercury. The temperature (in Celsius) of the spaceship's hull at the point (x, y, z) is given by

$$T(x,y,z) = 10e^{xy+2z},$$

where x, y, and z are all measured in millions of kilometers (Gigameters). The ship is currently at $(1, 2, 1)$.

(a) In what direction should the spaceship travel in order to experience the fastest *decrease* in temperature? (Express this direction as a unit vector.)

(b) What is the rate of change of the temperature if the ship proceeds in that direction?

7. (10 points) Let $f(x, y) = y\cos(x - y)$.

(a) Find the linear approximation of $f(x, y)$ around the point $(2, 2)$ and use it to estimate $f(2.1, 1.9)$.

(b) Write down the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(2, 2)$.

8. (10 points) Using Lagrange Multipliers, find the maximum and minimum values of $f(x, y) = x^2 + y^2 - 2y$, subject to the constraint $x^2 + 2y^2 = 8$. **No credit** will be given for a solution that does not make use of Lagrange Multipliers.

9. (15 points) Find all critical points of the function $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$ and characterize each one, if possible, as a local maximum, a local minimum, or a saddle point.

End of Exam

There are 3 types of people in the world.

Those who can count and those who can't.