Math 13 | Tufts University | April 11, 2011

Department of Mathematics

Exam 2

No calculators, notes, scratch paper or books are allowed. You must show all your work in your blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. **Simplify** *your answers as much as possible. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.*

1. (5 points) Using symmetry, determine if the following integrals are *positive, negative*, or *zero* for the regions shown in Figures 1 and 2. (You don't need to justify your answer.)

   (a) $\displaystyle\iint_R x^2 y\; dA$

   (b) $\displaystyle\iint_S \cos x \sin y\; dA$

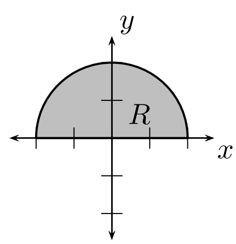

Figure 1: Problem 1a, region R

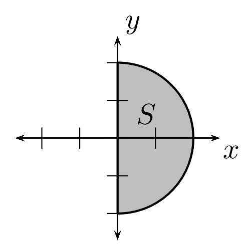

Figure 2: Problem 1b, region S

2. (5 points) Sketch the surface whose equation in cylindrical coordinates is given by $z = r^2$.

3. (5 points) Let D be the region contained inside the circle of radius $\sqrt{2}$ centered at the origin, above the line $y = x$, and in the first quadrant. (See Figure 3.) Express the integral

$$\iint_D x^2\; dA$$

as an iterated integral in *rectangular* coordinates. ***Do not evaluate***. (Hint: one order is much simpler.)

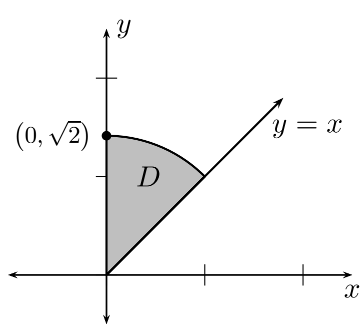

Figure 3: Problem 3

Exam continues

4. (15 points) Use the method of Lagrange multipliers to find the maximum value of the function

$$f(x, y, z) = x^2yz$$

on the surface with equation

$$x^2 + 2y^2 + z^4 = 7.$$

No credit will be given for a solution that does not use Lagrange multipliers!

5. (15 points) Consider the integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} \left(x^2 + y^2 + z^2\right)\, dz\, dy\, dx.$$

(a) Rewrite the integral as an iterated integral in *cylindrical coordinates.*

(b) Rewrite the integral as an iterated integral in *spherical coordinates.*

Do not evaluate *either of the above.* Sketches are *not* required; however sketches of the region and its projection to the xy-plane will help you receive partial credit for any incorrect answers.

6. (10 points) Figure 4 shows the solid bounded by the surfaces

$$x = 0, \quad z = 0, \quad y = 2 - z, \quad \text{and} \quad y = \sqrt{x}.$$

Set up iterated *triple* integrals that yield the volume of the solid in the following orders. ***Do not evaluate!***

(a) $dz\ dy\ dx$

(b) $dx\ dz\ dy$

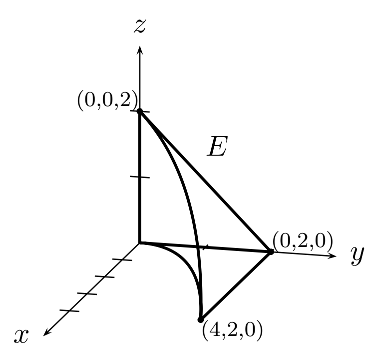

Figure 4: Problem 6

Exam continues

7. (15 points) Figure 5 shows a solid E. The solid is contained in the first octant, below the cone $z = \sqrt{x^2 + y^2}$, and inside the sphere $x^2 + y^2 + z^2 = 4$. Using spherical coordinates, evaluate the integral

$$\iiint_E z \, dV.$$

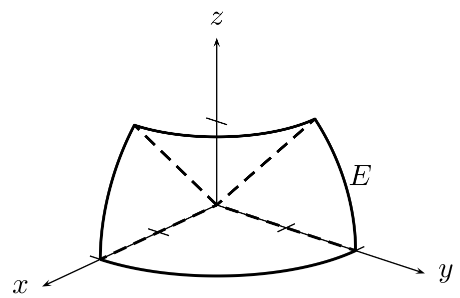

Figure 5: Problem 7

8. (10 points) Evaluate the line integral

$$\int_C x \, ds$$

where C is the curve in the xy-plane parametrized by $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j}$, for $0 \le t \le 2$.

9. (10 points) Evaluate the line integral

$$\int_C x^2 \, dx + 2 \, dy$$

where C is the semicircle of radius 2 shown in Figure 6.

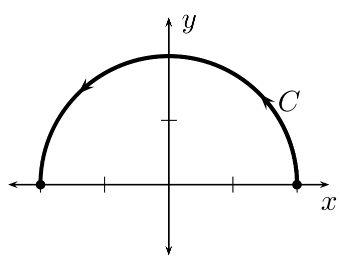

Figure 6: Problem 9

10. (10 points) The vector field $\mathbf{F}(x, y, z) = 2\sin y\,\mathbf{i} + (2x\cos y - z^3 y)\,\mathbf{j} - \frac{3}{2}z^2y^2\,\mathbf{k}$ is conservative.

    (a) Find a potential function for the vector field $\mathbf{F}(x, y, z)$.

    (b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve parametrized by

$$\mathbf{r}(t) = t^2\,\mathbf{i} + t\,\mathbf{j} + \cos t\,\mathbf{k}, \qquad 0 \le t \le \pi.$$

End of Exam