Math 13 | Tufts University | February 28, 2011

Department of Mathematics

Midterm Exam 1

No calculators, notes, scratch paper or books are allowed. You must show all your work in your blue book in order to receive a full credit. A correct answer with no work may not necessarily score any points. **Simplify** *your answers as much as possible. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.*

1. (10 points): Consider the three points in space

$$P(1,1,1),\ Q(1,2,3),\ \text{and}\ R(2,3,1).$$

   (a) Find the area of the triangle ΔPQR.

   (b) Find an equation of the plane containing the points P, Q, and R.

2. (15 points): Consider the two points $P(1,0,1)$ and $Q(5,0,0)$.

   (a) Let $\mathbf{a} = \langle 1,2,2\rangle$ and $\mathbf{b} = \overrightarrow{PQ}$. Find $\text{proj}_{\mathbf{a}}\mathbf{b}$.

   (b) Find the distance from the point P to the plane $x + 2y + 2z = 5$.

3. (5 points): Write the definite integral which represents the arc length of the space curve given by the vector valued function

$$\mathbf{r}(t) = \langle 2t - \sin 2t, \cos 2t\rangle, \quad 0 \le t \le \pi/4.$$

   Do NOT evaluate the integral.

4. (10 points): A particle moving in space has acceleration

$$\mathbf{a}(t) = 12t\ \mathbf{i} + 12t^2\ \mathbf{j} + 60t^3\ \mathbf{k}.$$

   Its initial velocity is

$$\mathbf{v}(0) = \mathbf{i} - \mathbf{k}$$

   and its initial position is

$$\mathbf{r}(0) = \mathbf{i}.$$

   Find the position $\mathbf{r}(t)$ of the particle at time t.

The exam continues on the other side.

5. (15 points): Let $f(x, y) = x^2 + 2x \ln y$.

   (a) Find the linear approximation of the function $f(x, y)$ at the point $(x, y) = (2, 1)$.

   (b) Use your answer in Part (a) to approximate the value of $f(2.1, 1.1)$.

6. (15 points):

   (a) The equation
   $$x^2yz - z^3 = 3$$
   defines z implicitly as a function of x and y. Find $\partial z/\partial x$ when $x = 2$, $y = 1$, and $z = 1$.

   (b) Suppose that the temperature at any point (x, y, z) in space is given by the function $T(x, y, z) = x^3y^2z$. Find the directional derivative of $T(x, y, z)$ at the point $P(1, -1, 1)$ in the direction from P to the point $Q(3, 1, 2)$.

   (c) Let $T(x, y, z)$ be the temperature function in Part (b) above. Find a *unit vector* **u** in the direction of which the temperature *increases fastest* at the point $P(1, -1, 1)$.

7. (10 points):

   (a) Find all the critical points of the function
   $$f(x, y) = 2x^2 + y^2 - 4x^2y.$$

   (b) Consider the function $g(x, y) = x^5 - 5xy + y^5$. Its critical points are $(0, 0)$ and $(1, 1)$. Determine whether each of these critical points gives a local minimum, a local maximum, or a saddle point of $g(x, y)$.

8. (10 points): Find an equation of the tangent plane to the surface
   $$x^2 + 2y^3 + 3z^4 = 28$$
   at the point $(3, 2, 1)$.

9. (10 points): Let $f(x, y) = x^2 + y^2 + x$.

   (a) Find all the critical points of $f(x, y)$.

   (b) Find the maximum value of $f(x, y)$ on the unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$.

End of exam.