Math 13	Tufts University Department of Mathematics Final Exam	May 10, 2010 8:30 to 10:30 am

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (20 points) Compute the following.
   (a) The area of the parallelogram formed by the vectors $\mathbf{i}+\mathbf{j}$ and $-\mathbf{i}+\mathbf{j}+\mathbf{k}$.
   (b) An equation for the plane containing the point $(0,4,2)$ that is parallel to the plane $x-5y+2z=2159$.
   (c) $\mathbf{a}\cdot\mathbf{b}$ where $|\mathbf{a}|=3$, $|\mathbf{b}|=3$ and the vectors $\mathbf{a}$ and $\mathbf{b}$ form an angle of $\frac{3\pi}{4}$.
   (d) $\dfrac{dz}{dx}$ at the point $(1,1,0)$ where z is defined implicitly via the equation
   $$\ln(xy+4z)=e^{-z}-1.$$
   (e) The directional derivative of $f(x,y)=\sin(xy)$ at $(\pi,1)$, in the direction of $\frac{1}{\sqrt{2}}\mathbf{i}+\frac{1}{\sqrt{2}}\mathbf{j}$.
2. (10 points) Let $f(x,y)=x^3+y^3$ be defined on the xy-region $D=\{(x,y)\mid x^2+y^2\leq 4\}$. Find the absolute maximum and absolute minimum values of f on D and the coordinates where they occur.
3. (10 points) Consider the integral $\int_0^2\int_0^{\sqrt{4-y^2}} xy\,dx\,dy$.
   (a) Sketch the region of integration.
   (b) Evaluate the integral by switching to polar coordinates.
4. (10 points) Consider the solid volume that lies inside both the sphere $x^2+y^2+z^2=4$ and the cylinder $x^2+y^2=1$.
   (a) Give a **triple** integral in **rectangular** coordinates for the volume of this solid. **Do NOT evaluate this integral!**
   (b) Change your integral from part (a) to **cylindrical** coordinates, and **compute** the volume.
5. (10 points) Evaluate the surface integral, $\iint_S xz\,dS$, where S is the surface with parametric equations $x=u$, $y=u\sin v$, $z=u\cos v$, for $0\leq u\leq 2$, $0\leq v\leq\frac{\pi}{2}$.
6. (10 points) Let $\mathbf{F}=\langle e^y+\sin z,\ xe^y,\ x\cos z\rangle$.
   (a) Show that $\mathbf{F}$ is conservative.
   (b) Find a potential function f such that $\nabla f=\mathbf{F}$.
   (c) **Without** explicitly computing an integral, find $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the curve parameterized by $\mathbf{r}(t)=\langle t,t^2,t+1\rangle$, $0\leq t\leq 1$.

The exam continues on the back!

7. (10 points) Let $\mathbf{F} = \langle e^{x^2 \sin x} + y^2, x + \ln(\cos y) \rangle$ and let C be the boundary curve of the unit circle $x^2 + y^2 = 1$, oriented positively. Compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$.

8. (10 points) Let C be a simple closed curve contained in the plane $x+2y+2z = 5$ that encloses a region of area 3 and is oriented counterclockwise when viewed from above. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = z\mathbf{i} + x\mathbf{j} - y\mathbf{k}$. (*Hint*: First explain how you can use Stokes' Theorem to compute this integral.)

9. (10 points) Let S be the surface of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $z = 4$, oriented outwards. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for $\mathbf{F} = (xy^2 + \cos(z^3))\mathbf{i} + (yx^2 + x^4)\mathbf{j} + z^2\mathbf{k}$.

End of Exam