Tufts University

Math 13 | **Department of Mathematics** | **April 5, 2010**

Exam 2 | **12:00 noon to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (12 points)

   (a) The volume of the ellipsoid
   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
   is given by the equation
   $$V(a, b, c) = \frac{4}{3}\pi abc.$$
   If a increases at a rate of 2 units per second, b decreases at a rate of 1 unit per second, and c is held constant, at what rate is the volume of the ellipsoid changing when $a = 5, b = 5$ and $c = 3$.

   (b) Let $f(x, y, z) = \frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{9}$. Find an equation for the tangent plane to the level surface $f(x, y, z) = 1$ at $(x_0, y_0, z_0) = (3, 4, 0)$.

2. (12 points) Find and classify (as local minima, local maxima, or saddle points) all critical points of $f(x, y) = e^{-x}(x^2 - y^2)$.

3. (12 points) Use the Method of Lagrange Multipliers to find three positive numbers whose sum is 300 and whose product is maximal. *No credit will be given for a solution that does not use Lagrange Multipliers!*

4. (12 points)*Set up and evaluate* a **double** integral for the volume of the region in $\mathbb{R}^3$ above the rectangle $\left[0, \frac{\pi}{2}\right] \times [0, 2]$ and below the graph $z = \frac{y}{2} \cos x \sin x$.

5. (12 points) Consider the integral $\displaystyle\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx$.

   (a) Sketch the region of integration.

   (b) Evaluate the integral by reversing the order of integration.

6. (12 points) Let E be the wedge in the first octant bounded by the cylinder $x^2 + y^2 = 1$ and by the planes $x = z$, $y = 0$ and $z = 0$.

   (a) *Set up* a **triple** integral in rectangular coordinates, in the order $dz\ dy\ dx$, for the volume of E. **Do** *not* **evaluate this integral.**

   (b) Convert your integral from part (a) to cylindrical coordinates and **compute the volume** of E.

The exam continues on the back!

7. (12 points) Let E be the solid inside the sphere $x^2 + y^2 + z^2 = 1$ and below the cone $z = \sqrt{x^2 + y^2}$. Use spherical coordinates to **compute the volume** of E.

8. (16 points) *Evaluate* the following line integrals

   (a)
   $$\int_C x^3\ dx + xy\ dy$$
   where C is the top half ($y \geq 0$) of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(-2, 0)$.

   (b)
   $$\int_C \mathbf{F} \cdot d\mathbf{r}$$
   where $\mathbf{F}(x, y, z) = y\mathbf{i} + (x + z)\mathbf{j} + x^2y\mathbf{k}$ and C is the curve parameterized by $\mathbf{r}(t) = \langle 2t, t^3, t^4 \rangle$ for $0 \leq t \leq 1$.

End of Exam