Tufts University

Math 13 **Department of Mathematics** **February 22, 2010**

Exam 1 **12:00 noon to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) **True or False - No Partial Credit**: On the first page of your blue book, answer the following questions as **True** or **False**.

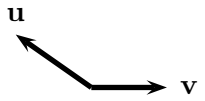

Figure 1: Problem 1

   (a) For $\mathbf{u}$ and $\mathbf{v}$ as shown in Figure 1, the vector $\mathbf{u} \times \mathbf{v}$ points into the page.
   (b) For $\mathbf{u}$ and $\mathbf{v}$ as shown in Figure 1, $\mathbf{u} \cdot \mathbf{v} > 0$.
   (c) $\mathbf{u} \cdot \mathbf{w} = 0$ implies that $\mathbf{u}$ and $\mathbf{w}$ are orthogonal.
   (d) The vectors $< -4, 6, -10 >$ and $< 6, -9, 15 >$ are parallel.
   (e) If $\mathbf{a}$ and $\mathbf{b}$ are parallel then $|\mathbf{a} \times \mathbf{b}| = 0$.

2. (12 points)

   (a) If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ find $\mathbf{a} \times \mathbf{b}$.
   (b) Find the scalar projection of $\mathbf{v}$ onto $\mathbf{u}$, $\text{comp}_{\mathbf{u}}\mathbf{v}$, for $\mathbf{u} =< 2, 1 >$ and $\mathbf{v} =< 1, 3 >$
   (c) Find the angle between the vectors $< 1, 2, -1 >$ and $< 2, 1, 1 >$.

3. (12 points) Consider the plane $\mathcal{P}$ given by $4x - 2y + 3z = 5$

   (a) For what value(s) of b is the line with parametric equations $x(t) = 1 - 2t$, $y(t) = 4 - t$ and $z(t) = 5 + bt$ perpendicular to the normal vector of $\mathcal{P}$?
   (b) At what point does the line with parametric equations $x(t) = 2t$, $y(t) = -1 + t$, and $z(t) = 1 - t$ intersect the plane $\mathcal{P}$?
   (c) Find an equation for the plane that includes the point $(2, -3, 4)$ and is parallel to $\mathcal{P}$.

4. (10 points) Consider the surface $\mathcal{S} : y^2 = 4x^2 + z^2$.

   (a) Draw a 2-dimensional sketch that shows the trace of $\mathcal{S}$ in the plane $z = 0$.
   (b) Draw a 2-dimensional sketch that shows the trace of $\mathcal{S}$ in the plane $x = 1$.
   (c) Draw a 2-dimensional sketch that shows the trace of $\mathcal{S}$ in the plane $y = 1$.
   (d) Identify $\mathcal{S}$ and draw a 3-dimensional sketch of the surface.

The exam continues on the back!

5. (10 points) Consider the space curve

$$\mathbf{r}(t) = (2t + \cos t)\mathbf{i} + (3e^{2t})\mathbf{j} + (1 + \tan t)\mathbf{k}$$

   (a) Find parametric equations for the line tangent to this curve at the point $(1, 3, 1)$.

   (b) Find the unit tangent vector, $\mathbf{T}(t)$, to $\mathbf{r}(t)$ at $t = 0$.

6. (12 points) Let $\mathbf{r}(t) = \langle \frac{1}{2}\cos(t^2), \frac{1}{2}\sin(t^2), t^2 \rangle$. Reparametrize $\mathbf{r}(t)$ with respect to its arc length, as measured from the point where $t = 0$ in the direction of increasing t.

7. (10 points) Consider the function $f(x, y) = 3x^2 + y^2$.

   (a) What are the domain and range of f?

   (b) Sketch and label the level curves, $f(x, y) = k$, for $k = 1, 3, 12$.

   (c) Find the equation for the tangent plane to $f(x, y)$ at the point $(1, 1, 4)$.

8. (12 points) Define the function $z(x, y)$ implicitly by the equation

$$\cos(xyz) = xy + yz.$$

   Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ using implicit differentiation. Evaluate these at the point $(0, 1, 1)$.

9. (12 points) The net velocity change, V, of a spacecraft with propellant mass M and payload mass P is given by the equation

$$V(M, P) = R \ln\left(\frac{M + P}{P}\right),$$

   where R is the (constant) velocity of the rocket exhaust.

   Use differentials to estimate the maximum error in the calculated net velocity change if the propellant and payload masses of a spacecraft are measured to be $M = 2000$ kg and $P = 100$ kg, with possible errors of as much as 1% in each measurement. You need not simplify any natural logs that appear in your answer.

End of Exam