	Tufts University	Bluebook No. ____________
Math 13	Final Examination	May 4, 2009
	(100 points)	8:30 a.m. – 10:30 a.m.

Instructions: Write your bluebook number in the space above. No calculators, notes or books are allowed. Except for the True-False questions, you should show all your work in order to receive full credit. Simplify your answers. Please box your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

1. (10 points) True/False – No partial credit will be given. Write your answers to this question on the inside front cover of your blue book. Write **T** if the statement is always true, and write **F** otherwise.

   (a) The surface in space with the equation $x^2 + y^2 = 1$ is a circle.

   (b) For all vectors $\vec{v}_1$ and $\vec{v}_2$ in space, $\vec{v}_1 \cdot (\vec{v}_1 \times \vec{v}_2) = 0$.

   (c) Let $f(x, y, z)$ be a function of 3 variables and let p be a point such that the first partials of f at p exist. Then $\vec{\nabla} f(p)$ is always a unit vector.

   (d) If a differentiable function $f(x, y)$ is such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$, then f has a local maximum or a local minimum at (a, b).

   (e) If $\vec{F}$ is a vector field in space, then $\operatorname{div} \vec{\operatorname{curl}} \vec{F} = 0$.

2. (10 points)

   (a) Let $\mathcal{P}$ be the plane that contains the origin and the line whose parametric equations are $x = 3 + t$, $y = 2t$, and $z = 2 - t$. Find parametric equations of the normal line to $\mathcal{P}$ through the point $(3, 0, 2)$.

   (b) Find an equation of the tangent plane to the surface $e^x \ln y + z^3 = 8$ at the point $(3, 1, 2)$.

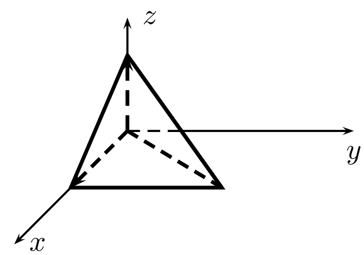
3. (5 points) Describe the level surfaces of the function $f(x, y, z) = x^2 - y^2 - z^2$ for $k = -1, 0, 1$. You may choose to either name the surfaces or sketch them.

4. (10 points) Let $\mathcal{E}$ be the solid bounded by the paraboloid $z = 1 + x^2 + y^2$, the xy-plane, and the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

   (a) Express the volume of $\mathcal{E}$ as an iterated **double** integral in **polar** coordinates.

   (b) Compute the integral in (a).

5. (10 points) Let $\mathcal{E}$ be the solid tetrahedron in the first octant bounded by the xz-plane, the xy-plane, and the planes $x + z = 1$, $y = 2x$. Set up the iterated triple integral for the volume of $\mathcal{E}$ in the order $dx\,dy\,dz$. DO NOT EVALUATE.

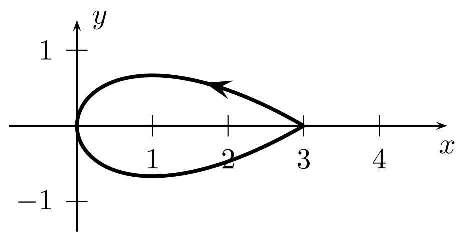


(Exam continues on reverse side.)

6. (10 points) Use the Green's Theorem area formula (Area $= \oint_C x\,dy$) to compute the area of the region enclosed by the curve

$$\vec{r}(t) = t^2\,\vec{\imath} + \left(\frac{t^3}{3} - t\right)\vec{\jmath},$$

where $-\sqrt{3} \le t \le \sqrt{3}$.



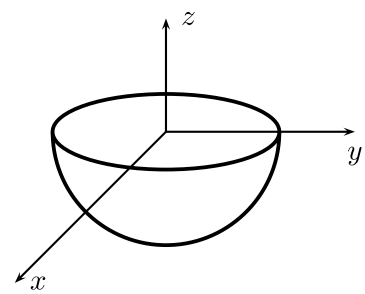
7. (10 points) Find the length of the curve $\vec{r}(t) = \langle t, \frac{1}{3}(t^2+2)^{3/2}\rangle$ from $t = 0$ to $t = 1$.

8. (15 points) Let $\mathcal{S}$ be the lower half of the unit sphere $x^2 + y^2 + z^2 = 1$.

   (a) Parametrize $\mathcal{S}$ using spherical coordinates. Be sure to include the limits for the parameters.

   (b) Parametrize $\mathcal{S}$ as the graph of a function $z = g(x, y)$. Be sure to describe the domain of the parameters x, y.

   (c) Use (a) or (b) to compute $\iint_{\mathcal{S}} z\,dS$. If you use (a), you may use the fact that $|\vec{r}_\phi \times \vec{r}_\theta| = \sin\phi$.



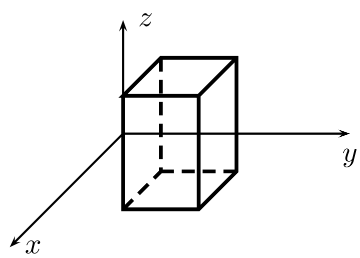
9. (10 points) Let $\mathcal{S}$ be the surface of the solid E given by

$$0 \le x \le 2, \quad 1 \le y \le 3, \quad -1 \le z \le 2$$

with outward orientation. A fluid flow in space has velocity vector field

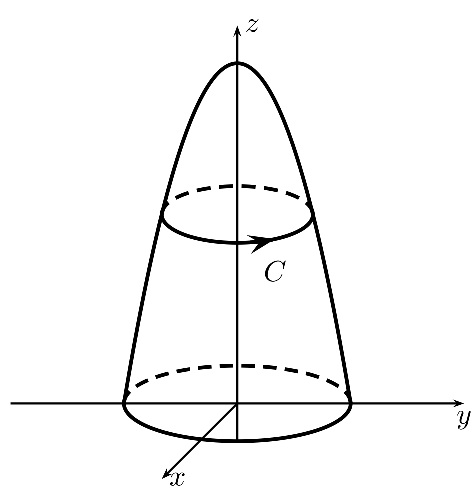
$$\vec{F} = \langle x + e^{y^2 \tan z},\ 3xe^{xz},\ \cos y + 2z\rangle.$$

Use the divergence theorem to find the outward flux $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$ of $\vec{F}$ over $\mathcal{S}$.



10. (10 points) Let C be the curve of intersection of the plane $z = 5$ and the paraboloid $z = 9 - x^2 - y^2$, oriented counterclockwise as viewed from above. Use Stokes' Theorem to evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle 2xz,\ x^3/3,\ 5\rangle$.

(Hint. $\cos^2\theta = (1 + \cos 2\theta)/2$.)



(End of Exam)