Tufts University

Math 13	**Department of Mathematics**	**April 6, 2009**
	Exam 2	**12 to 1:20 p.m.**

Instructions: No calculators, notes or books are allowed. You should show all work to receive full credit. **Simplify your answers.** Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

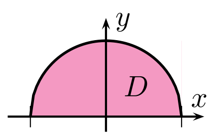
1. (10 pts) True/False — No Partial Credit. On the inside front cover of your blue book answer the following questions as either **True** or **False**.

   a) The point $(x, y, z) = (3\sqrt{3}, 3, 5)$ in Cartesian coordinates equals $(r, \theta, z) = (6, \frac{\pi}{6}, 5)$ in cylindrical coordinates.

   b) The equation of a sphere centered at the point $(0, 0, 4)$ and radius 4 in spherical coordinates is $\rho = 4$.

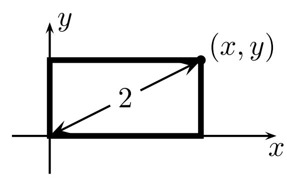
   c) The vector field $\vec{F}(x, y) = \langle \cos y, \sin x \rangle$ is conservative.

   d) If D is the region $\{(x, y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1, y \geq 0\}$ in the plane, then $\iint_D \sin(x^3 y)\, dA = 0$.

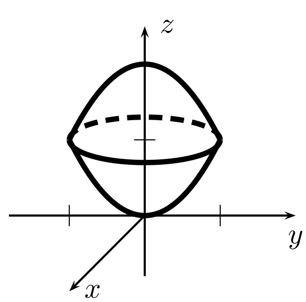


   e) If $\int_C \vec{F} \cdot d\vec{r}$ is independent of path, then $\int_C \vec{F} \cdot d\vec{r} = 0$ whenever C is a closed path.

2. (15 pts) Using Lagrange multipliers, set up and solve the system of equations associated with finding the dimensions of the rectangle of largest area if the length of its diagonal is 2. To receive full credit, you must use Lagrange multipliers and you must show all your work. (Hint: Instead of the diagonal, consider the square of the diagonal.)



3. (10 pts) Set up an iterated triple integral in cylindrical coordinates for the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the paraboloid $z = 2 - x^2 - y^2$. DO NOT EVALUATE.



(Exam continues on reverse side.)

4. (10 pts) Evaluate the following double integral (*Hint*: You may have to change the order of integration):

$$\int_0^4 \int_{x/2}^2 e^{y^2}\,dy\,dx.$$

5. (10 pts) Rewrite the following integral as a triple integral in spherical coordinates:

$$\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} \int_0^{\sqrt{16-x^2-y^2}} y\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy.$$

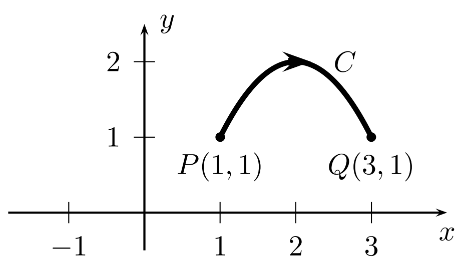
DO NOT EVALUATE.

6. (10 pts) Evaluate the line integral $\int_C (x+y)^4\,ds$, where C is the line segment from the point $A(-1,2)$ to the point $B(7,-4)$.

7. (15 pts) Let $\vec{F} = \langle z, e^y, -x\rangle$ be a vector field and C the path with vector equation $\vec{r}(t) = \sin(2t)\vec{\imath} + t^2\vec{\jmath} + \cos(2t)\vec{k}$ for $t \in [0,1]$. Evaluate $\int_C \vec{F}\cdot d\vec{r}$.

8. (10 pts) Consider the vector field $\vec{F}(x,y) = (\frac{1}{x} + ye^{xy})\vec{\imath} + (\frac{1}{y} + xe^{xy})\vec{\jmath}$ in the first quadrant of the plane.

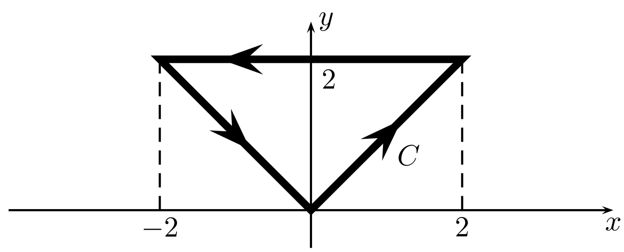
   a) Find a function f such that $\vec{F} = \vec{\nabla} f$.
   b) Under the action of the force $\vec{F}$, an object moves along the curve $C : y = -x^2+4x-2$ from the point $P(1,1)$ to the point $Q(3,1)$. Use the fundamental theorem for line integrals to calculate $\int_C \vec{F}\cdot d\vec{r}$, the work done along this curve.



9. (10 pts) Consider the vector field $\vec{F}(x,y) = \left(2y + 3x^2y\right)\vec{\imath} + x^3\vec{\jmath}$. Use Green's Theorem to evaluate

$$\int_C \vec{F}\cdot d\vec{r}$$

where C is the path shown below. (There is no credit if you do not use Green's theorem.)



End of Exam