Math 42 | Tufts University | November 10, 2014

Tufts University
Department of Mathematics
Exam 2

Math 42 — **November 10, 2014** — **12:00 pm to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) **True or False - No Partial Credit**: On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) If $\nabla f(x,y,z) \neq \mathbf{0}$, the direction that yields the largest directional derivative of f at (x,y,z) is $\frac{\nabla f(x,y,z)}{|\nabla f(x,y,z)|}$.

   (b) If $D = \{(x,y) \mid h_1(y) \leq x \leq h_2(y),\ c \leq y \leq d\}$, then the area of D is given by $\displaystyle\int_{h_1(y)}^{h_2(y)} \int_c^d 1\, dy\, dx$.

   (c) Suppose D is an open simply-connected region containing the unit circle, C. Let $F(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$, such that $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ for all points in D, then $\int\limits_C \mathbf{F}(x,y) \cdot d\mathbf{r} = 0$.

   (d) The vector field $\mathbf{F}(x,y,z) = \langle x^2, y, z \rangle$ is conservative on $\mathbb{R}^3$.

   (e) $f(x,y,z) = xz + x^2y + \sin(y)$ is a potential function for the vector field, $\mathbf{F}(x,y,z) = \langle 2xy + z, x^2 + \cos(y), x \rangle$.

2. (10 points) Suppose that $z = \sin(\pi x)\cos(\pi y)$, and $x = se^t$, $y = s + t$. Find $\frac{\partial z}{\partial t}$ when $s = 1$ and $t = 0$.

3. (10 points) Let $f(x,y) = 9x - 3x^3 - 12y + y^3$.

   (a) Find all critical points of $f(x,y)$ in the $xy-$plane.

   (b) Determine where $f(x,y)$ has a local maximum, local minimum, or a saddle point. Determine the values of f at these points as well.

4. (15 points) Using Lagrange multipliers, find the absolute maximum and minimum values of

$$f(x,y,z) = 2x + 4y - 12z,$$

   on the ellipse $x^2 + 2y^2 + z^2 = 39$.

The exam continues on the back!

5. (15 points) The area inside the circle $(x-a)^2 + y^2 = a^2$ is πa^2, since this is a circle of radius a.

   (a) Sketch the circle and, using inequalities, express the region, D, inside the circle in **both** Cartesian **and** polar coordinates.

   (b) Set up, **but DO NOT EVALUATE**, the integral for computing the area, $\iint_D 1\, dA$, in Cartesian coordinates.

   (c) Set up, **but DO NOT EVALUATE**, the integral for computing the area in polar coordinates.

6. (15 points) Consider the solid region that is bounded by the 4 planes, $x = 0$, $y = 0$, $z = 0$, and $x + 2y + z = 2$. Denote this region by E.

   a. Draw the solid region, E, clearly labeling the bounding planes and any intersection points with the coordinate axes.

   b. Set up, **but DO NOT EVALUATE**, the triple integral of $f(x, y, z) = z$ over the region E using the following order of integrations:

      i. $dzdydx$

      ii. $dydxdz$

      iii. $dxdydz$

7. (15 points) Consider the region bounded above by the sphere, $x^2 + y^2 + z^2 = 3$, and below by the cone, $z = \sqrt{3(x^2 + y^2)}$.

   (a) Set up but **DO NOT EVALUATE** the triple integral representing the volume of the region using cylindrical coordinates (**Hint:** The intersection of the sphere and the cone occurs in the plane $z = \frac{3}{2}$).

   (b) Evaluate the triple integral using spherical coordinates.

8. (10 points) Consider the line segment, C, joining the origin and the point $(1, 1, 1)$. Let $f(x, y, z) = x - 3y^2 + z$.

   (a) Integrate $f(x, y, z)$ over C, $\int\limits_C f(x, y, z)\, ds$.

   (b) Find the line integral of the gradient field, ∇f, over the same curve, C, $\int\limits_C \nabla f \cdot d\mathbf{r}$.

End of Exam