Tufts University

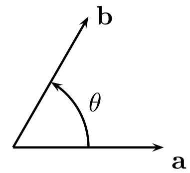
Math 42 — Department of Mathematics — September 29, 2014

Exam 1 — Noon–1:20 p.m.

Instructions: Do all nine problems below. Except for Problem 1, you must show your work and justify your answers in order for partial or full credit to be awarded. No books, notes, calculators, or mobile phones are allowed during this exam. *You are required to sign each examination blue book that you are handing in. With your signature, you are pledging that you have neither given nor received any help pertaining to this exam. If you are found in violation of this policy, you will be refered to the Dean of Students and automatically receive an* **F** *for the course.*

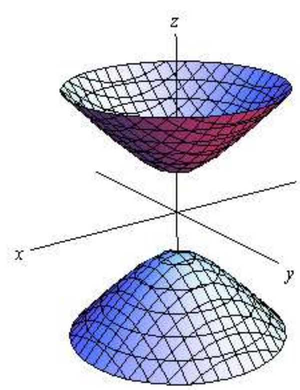
1. (10 points) **True or False – no partial credit.** On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) The vector $\mathbf{c}$ defined by $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ lies in a direction perpendicular to this page, pointing into the page and away from you. (See the figure below.)



   (b) The line $x = 1 + t,\ y = -2t,\ z = 3 + t$ is parallel to the plane $x + 2y + 3z = 10$.

   (c) The surface below is the graph of the equation $x^2 + y^2 = z^2 + 1$.



   (d) The curve $\mathbf{r}(t) = \frac{2}{3}\cos t\,\mathbf{i} - \frac{2}{3}\sin t\,\mathbf{j} - \frac{1}{3}t\,\mathbf{k}$ uses arc length as a parameter.

   (e) The lines $y = x$ and $y = -x$ are level curves of the surface $f(x, y) = x^2 - y^2 + 1$.

2. (10 points) Let $\mathbf{u} = 2\,\mathbf{i} + 3\,\mathbf{j} + 3\,\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 2\,\mathbf{j} + 2\,\mathbf{k}$.

   (a) Find the scalar component $\text{scal}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ in the direction of $\mathbf{v}$.

   (b) Find the orthogonal projection $\text{proj}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ onto $\mathbf{v}$.

The exam continues on the opposite side of this sheet.

3. (15 points) Consider the triangle with vertices $P(1, 2, -1)$, $Q(3, 0, 2)$, and $R(4, 1, 0)$.

   (a) Find the area of the triangle ΔPQR.

   (b) Find an equation of the plane containing the triangle ΔPQR.

4. (10 points) Find parametric equations of the line of intersection of the two planes $x+y+z=1$ and $x-2y+3z=-4$. (You may use the fact that the point $P(1, 1, -1)$ lies in both planes.) You may also express your answer in terms of a vector function $\mathbf{r}(t) = \underline{\quad}\,\mathbf{i} + \underline{\quad}\,\mathbf{j} + \underline{\quad}\,\mathbf{k}$.

5. (10 points) A particle moves through space with acceleration vector $\mathbf{a}(t) = -10\,\mathbf{k}$. Its initial position is $\mathbf{r}(0) = 2\,\mathbf{i} + 3\,\mathbf{j}$ and its initial velocity is $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

   (a) Find its velocity vector $\mathbf{v}(t)$.

   (b) Find its position vector $\mathbf{r}(t)$.

6. (10 points) Let $x^2 - y^2 + z^2 = 1$. Draw four separate sketches, as described below.

   (a) Sketch the yz-trace.

   (b) Sketch the xy-trace.

   (c) Sketch the trace in any plane $y = k$.

   (d) Sketch the surface given by this equation.

7. (15 points) Let $f(x, y) = \ln(4 - x - y)$.

   (a) Describe the domain of $f(x, y)$, using inequalities.

   (b) Draw the region in the (x, y) plane corresponding to the domain of $f(x, y)$ that you obtained in Part (a).

   (c) Identify and sketch the level curves of $f(x, y)$ for $k = 0$, $k = \ln 2$, and $k = \ln 4$.

8. (10 points) Let $f(x, y) = e^x\,(\cos y + \sin y)$.

   (a) Compute the following second order partial derivatives of $f(x, y)$:

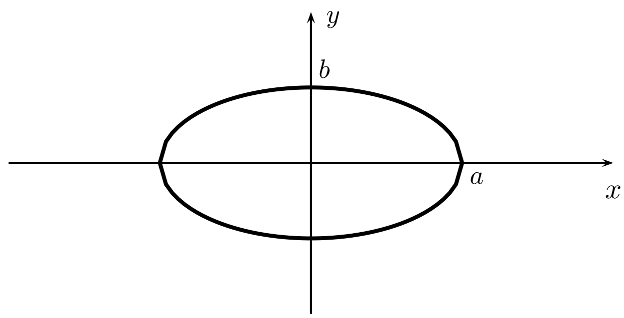
$$\frac{\partial^2 f}{\partial x^2}, \quad \frac{\partial^2 f}{\partial y^2}, \quad \text{and} \quad \frac{\partial^2 f}{\partial x\,\partial y}.$$

   (b) Show that $f(x, y)$ satisfies *Laplace's equation*

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

The exam continues on the next page.

9. (10 points) The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (shown in the figure below) can be described by the parametric equations $x = a\,\cos t,\ y = b\,\sin t$, for $0 \le t \le 2\pi$. Write down an integral representing the arc length (i.e., the circumference) of the entire ellipse. DO NOT EVALUATE.



End of Exam.