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| Math 42 | **Final Examination**<br>Dec. 14, 2012<br>8:30-10:30 AM | Fall, 2012 |

As voted by the math department: *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Show all your work and give reasons for your answers. No calculators, notes, or books are allowed on the test. Please ask if you want to know what you can assume on the test.

1. (15 points) For each item below, **box in** your answer. In (b) and (c), we will look *only* at your boxed-in answer.

   (a) Calculate the vector projection ($\text{proj}_{\vec{w}}\,\vec{v}$) of $\vec{v}$ in the direction of $\vec{w}$ , where

   $$\vec{v} = \vec{\imath} + \vec{\jmath} + \vec{k}$$
   $$\vec{w} = \vec{\imath} - 2\vec{\jmath} - \vec{k}.$$

   (b) Rewrite the iterated integral

   $$\int_0^2 \int_{\sqrt{x/2}}^1 (\sin x)\sqrt{1-y}\,dy\,dx$$

   with the order of integration reversed.

   (c) Set up (but do not attempt to evaluate) an integral expressing the length of the curve $\mathcal{C}$ given parametrically by

   $$x = 2\sin t \qquad y = 3\cos t \qquad z = \frac{t}{\pi}. \qquad 0 \le t \le 2\pi$$

2. (10 points) Find an equation for the plane containing the three points $P(1,0,-1)$, $Q(2,-1,0)$, and $R(3,2,-1)$.

3. (10 points) Find the absolute maximum and absolute minimum values of the function

   $$f(x,y,z) = x + 2y - 2z$$

   on the sphere

   $$x^2 + y^2 + z^2 = 9.$$

4. (10 points) The function

   $$f(x,y) = \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} - 2xy + \frac{y^2}{2}$$

   has critical points at $(0,0)$, $(1,2)$ and $(-3,-6)$. Analyze each of the *two* critical points $(0,0)$ and $(1,2)$ to determine whether it is a local minimum, a local maximum, or neither (*i.e.*, a saddle point). (*Don't bother with the third critical point.*)

5. (10 points) Consider the vector field

$$\overrightarrow{F}(x,y,z) = (x^2 - z^2)\overrightarrow{\imath} + (y^2 - x^2)\overrightarrow{\jmath} + (z^2 - y^2)\overrightarrow{k}.$$

(a) Calculate the divergence of $\overrightarrow{F}$, div $\overrightarrow{F} = \overrightarrow{\nabla} \cdot \overrightarrow{F}$.

(b) Calculate the curl of $\overrightarrow{F}$, curl$(\overrightarrow{F}) = \overrightarrow{\nabla} \times \overrightarrow{F}$.

6. (10 points) Let

$$f(x,y) = xy$$

and let $\mathcal{R}$ be the region in the plane (see Figure 1) bounded by the lines

$$\begin{aligned} y &= 0 \\ y &= x \\ y &= 2 - x. \end{aligned}$$

Set up and evaluate the double integral $\iint_{\mathcal{R}} f(x,y)\, dA$.

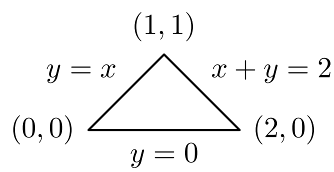


Figure 1: Problem 6

7. (13 points) Consider the vector field

$$\overrightarrow{F}(x,y) = (3x^2 - 2y + 2)\overrightarrow{\imath} + (2y - 2x + 1)\overrightarrow{\jmath}.$$

(a) Either show that $\overrightarrow{F}$ is *not* conservative, or find a potential function for $\overrightarrow{F}$.

(b) Evaluate the line integral

$$\int_{\mathcal{C}} \overrightarrow{F} \cdot \overrightarrow{T}\, ds = \int_{\mathcal{C}} \overrightarrow{F} \cdot d\vec{r}$$

where $\mathcal{C}$ is the path going in a straight line from $(-1, 0)$ to $(1, 0)$.

8. (12 points) Consider the planar vector field

$$\overrightarrow{F}(x,y) = (x - y)\overrightarrow{\imath} + (2x + y)\overrightarrow{\jmath}$$

and the closed, directed curve $\mathcal{C}$ consisting of the arc of the parabola $y = x^2$ from $(-2, 4)$ to $(2, 4)$ followed by the straight line segment from $(2, 4)$ to $(-2, 4)$ (See Figure 2).

(a) Set up an *iterated* (double) integral equal to the circulation integral

$$\oint_{\mathcal{C}} \overrightarrow{F} \cdot \overrightarrow{T}\, ds = \oint_{\mathcal{C}} \overrightarrow{F} \cdot d\vec{r}.$$

(b) Evaluate $\oint_{\mathcal{C}} \overrightarrow{F} \cdot \overrightarrow{T}\, ds$.

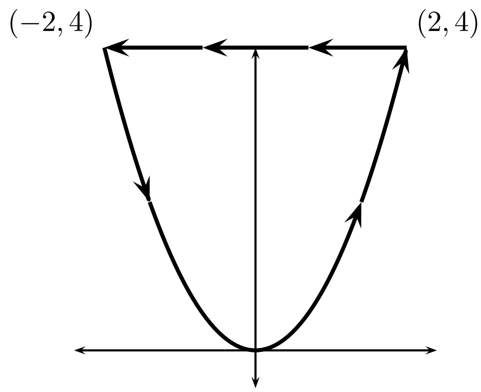


Figure 2: Problem 8

9. (10 points) Find the (outward) flux

$$\iint_{\mathcal{S}} \overrightarrow{F} \cdot \overrightarrow{n}\ d\mathcal{S} = \iint_{\mathcal{S}} \overrightarrow{F} \cdot d\overrightarrow{\mathcal{S}}$$

of the vector field

$$\overrightarrow{F}(x, y, z) = x\,\overrightarrow{\imath} + y\,\overrightarrow{\jmath} + z\,\overrightarrow{k}$$

over the boundary $\mathcal{S} = \partial\mathcal{D}$ of the solid $\mathcal{D}$ bounded below by the cone

$$z = \sqrt{x^2 + y^2}$$

and above by the plane

$$z = 2.$$

(See Figure 3.) (*Hint:* Note that the boundary consists of both the cone and the disc. You may evaluate the flux directly, but consider using the Divergence Theorem.)

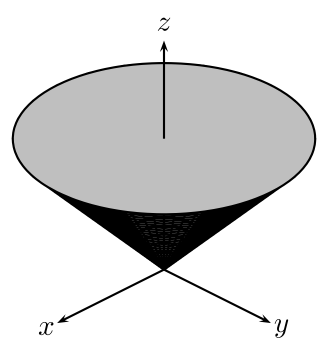


Figure 3: Problem 9

**End of exam**