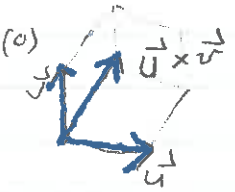
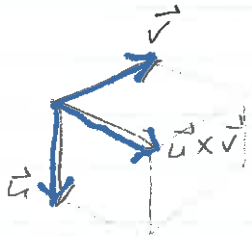
Math 42 Solutions to Exam 1 Fall 2012

1. (a) II
(b) VII
(c) IV
(d) III

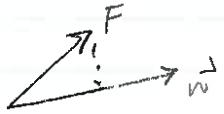
2. (a)





(b) $\cos\theta = \dfrac{\vec{F}\cdot\vec{w}}{|\vec{F}||\vec{w}|} = \dfrac{2\cdot 3 - 1\cdot 4}{\sqrt{5}\ 5} = \boxed{\dfrac{2}{5\sqrt{5}}}$

(c)



$\vec{v_1} = \text{proj}_{\vec{w}}\vec{F} = \dfrac{\vec{F}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \dfrac{2}{25}\begin{bmatrix}3\\4\end{bmatrix} = \boxed{\begin{bmatrix}6/25\\8/25\end{bmatrix}}$

$\vec{v_2} = \vec{F} - \vec{v_1} = \begin{bmatrix}2\\-1\end{bmatrix} - \begin{bmatrix}6/25\\8/25\end{bmatrix} = \boxed{\begin{bmatrix}44/25\\-33/25\end{bmatrix}}$.

3. $\boxed{\begin{aligned} x &= 2 - t \\ y &= 1 - 2t \\ z &= 1 + 3t \end{aligned}}$

4. $\vec{a} = \begin{bmatrix}0\\-32\end{bmatrix}$, $\vec{v}(0) = \begin{bmatrix}v_0\\0\end{bmatrix}$, $\vec{r}(0) = \begin{bmatrix}0\\400\end{bmatrix}$.

$\vec{v}(t) = \int \vec{a}(t)\,dt = \begin{bmatrix}c_1\\-32t\end{bmatrix} \Rightarrow c_1 = v_0$

$\vec{v}(t) = \begin{bmatrix}v_0\\-32t\end{bmatrix}$

$\vec{r}(t) = \begin{bmatrix}v_0 t + c_3\\-16t^2 + c_4\end{bmatrix} \Rightarrow \begin{bmatrix}c_3\\c_4\end{bmatrix} = \begin{bmatrix}0\\400\end{bmatrix}$

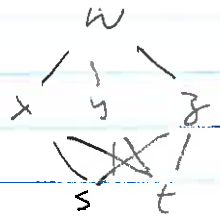
$\vec{r}(t) = \begin{bmatrix}v_0 t\\-16t^2 + 400\end{bmatrix}$

When the water hits the fire, $x = v_0 t = 3000$

$y = -16t^2 + 400 = 0 \Rightarrow t^2 = 25 \Rightarrow t = 5$

$v_0 t = v_0 5 = 3000 \Rightarrow v_0 = \boxed{600}$ ft/sec

5. (a)



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

(b) $3x^2 + 2y + 2x\frac{dy}{dx} - 4y^3\frac{dy}{dx} = 0$

$(2x - 4y^3)\frac{dy}{dx} = -3x^2 - 2y$

$\frac{dy}{dx} = \frac{-3x^2 - 2y}{2x - 4y^3}$

$= \frac{-(27+2)}{6-4} = \boxed{-\frac{29}{2}}$

6 $f_x = y\cos xy = 2 \cdot 0 = 0$

$f_y = 1 + x\cos xy = 1 + 0 = 1$

$z = 0 \cdot \left(x - \frac{\pi}{4}\right) + (y - 2) + 3$

$\boxed{z = y + 1}$

7. (a) $f_x = e^y = e^{\ln 4} = 4$

$f_y = xe^y = \frac{3}{4}e^{\ln 4} = 3$

$\vec{\nabla} f\left(\frac{3}{4}, \ln 4\right) = \boxed{\begin{bmatrix} 4 \\ 3 \end{bmatrix}}$

(b) $D_{\vec{u}} f = \vec{\nabla} f \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \boxed{-4}$

(c) $|\vec{\nabla} f| = \boxed{5}$

8. $f_x = 3x^2 - 3$ $\quad f_{xx} = 6x$

$f_y = 3y^2 - 3$ $\quad f_{yy} = 6y$

$f_{xy} = 0$

$D = \begin{vmatrix} 6x & 0 \\ 0 & 6y \end{vmatrix} = 36xy$

	$(1,1)$	$(-1,1)$
f_{xx}	6	-6
D	36	-36
	loc min	saddle point