Instructor's Name _____ Your Name _____

Math 42 Calculus III October 9, 2012

Exam 1 noon–1:20 p.m.

Instructions: No calculators, notes or books are allowed. You should show all work to receive full credit. **Simplify your answers.** Please box your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam. Write your answers to Questions 1, 2, and 3 on this exam; write the solutions to all other questions in the bluebook provided.

Write the answers to Questions 1, 2, an 3 on this exam. No work is needed in Questions 1 and 2(a), but show your work in the other questions.

1. (16 points) Match each of the following equations with its graph (labelled I to VII).

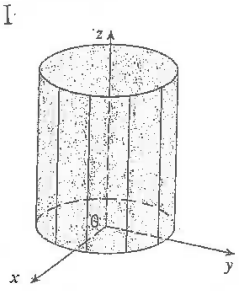
   (a) $y^2 + z^2 = 1$ _____

   (b) $z = y^2 - x^2$ _____

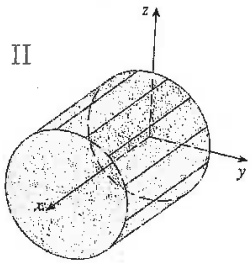
   (c) $\frac{x^2}{4} + y^2 - \frac{z^2}{9} = 1$ _____

   (d) $z = 4x^2 + y^2$ _____

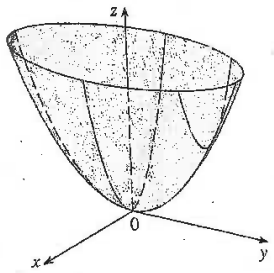
I



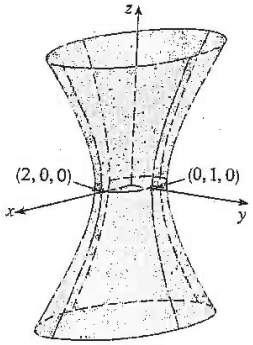
II



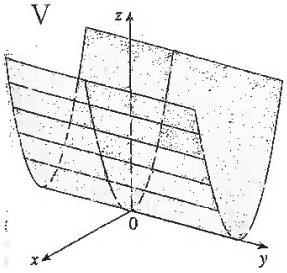
III



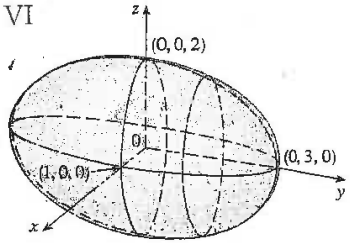
IV



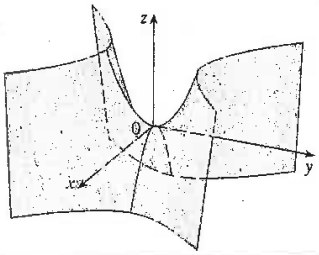
V



VI



VII



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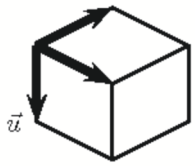
2. (20 points)

   (a) (4 points) In each of the following drawings, the three mutually perpendicular unit vectors shown are $\vec{u}$, $\vec{v}$, and $\vec{u} \times \vec{v}$. The vector $\vec{u}$ is already labelled; label the other two. These vectors are visible edges of the boxes drawn below.



   (b) (6 points) A force is given by $\vec{F} = 2\,\vec{\imath} - \vec{\jmath}$. Find $\cos\theta$, where θ is the angle between $\vec{F}$ and $\vec{w} = 3\,\vec{\imath} + 4\,\vec{\jmath}$.

   (c) (10 points) Decompose the force $\vec{F}$ in part (b) into components parallel and perpendicular to $\vec{w}$, that is, find vectors $\vec{v}_1$ and $\vec{v}_2$ such that $\vec{v}_1 + \vec{v}_2 = \vec{F}$, $\vec{v}_1$ is parallel to $\vec{w}$, and $\vec{v}_2$ is perpendicular to $\vec{w}$.

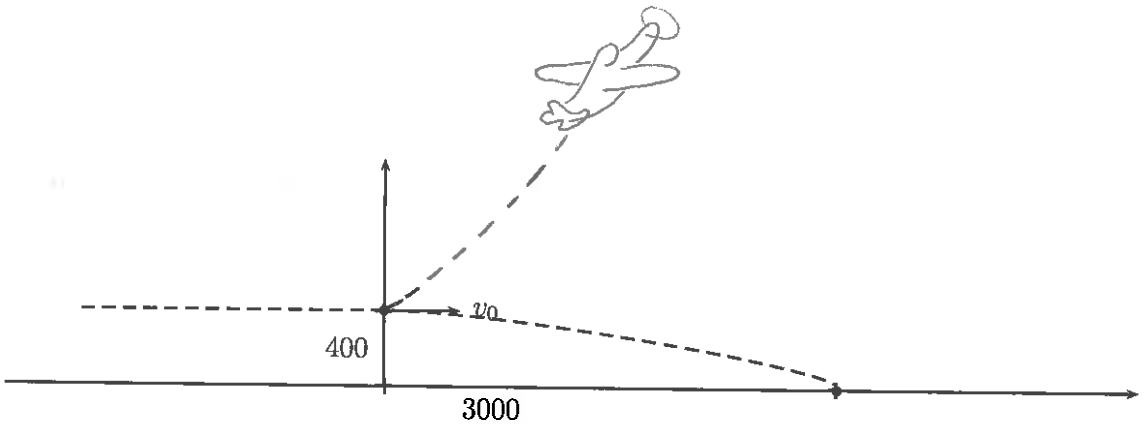
3. (5 points) Find parametric equations of the line through $(2, 1, 1)$ and parallel to the line given by

$$x = 5 - t, \quad y = 7 - 2t, \quad z = 9 + 3t.$$

(Exam continues on reverse side)

Write the solutions to Questions 4 through 8 in the bluebook provided. Show your work.

4. (10 points) A plane dropping water on a forest fire is forced by the heat and updraft to release its load at an altitude of 400 feet and a horizontal distance of 3000 feet from the fire (see sketch). If one assumes that the plane releases the water while flying horizontally, i.e., there is initially no vertical component of velocity, and that only gravity acts on the load (with downward acceleration $g = 32\ \mathrm{ft/s^2}$), at what horizontal speed v_0 must the plane release the water if it is to hit the fire?



5. (15 points) (a) (5 points) Suppose $w = F(x, y, z)$ and x, y, and z are functions of s and t. Write down the chain rule for $\partial w/\partial t$.

   (b) (10 points) Suppose $x^3 + 2xy - y^4 = 32$ defines y implicitly as a function of x near the point $(3, 1)$. Find dy/dx at $(3, 1)$.

6. (10 points) Find an equation of the tangent plane to the graph of the function $z = f(x, y) = y + \sin xy$ at the point $(\frac{\pi}{4}, 2, 3)$.

7. (12 points) Let $f(x, y) = xe^y$.

   (a) Compute the gradient $\vec{\nabla} f(\frac{3}{4}, \ln 4)$.

   (b) Compute the directional derivative of f at $(\frac{3}{4}, \ln 4)$ in the direction of the negative x-axis.

   (c) What is the maximum value of all directional derivatives of f at $(\frac{3}{4}, \ln 4)$?

8. (12 points) Two of the critical points of the function $f(x, y) = x^3 + y^3 - 3x - 3y$ occur at $(1, 1)$ and $(-1, 1)$. Classify the two points as local max, local min, saddle point, or inconclusive.

(End of Exam)