Tufts University

Math 13 | **Department of Mathematics** | **November 14, 2011**

Exam 1 | **12:00 noon to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) **True or False - No Partial Credit**: On the first page of your blue book, answer the following questions as **True** or **False**.
   (a) $\int_0^1 \int_0^x \sqrt{x+y^2}\,dy\,dx = \int_0^x \int_0^1 \sqrt{x+y^2}\,dx\,dy$
   (b) The point $(x,y,z) = (1, \frac{1}{\sqrt{3}}, 1)$ in Cartesian coordinates is the same as $r = \frac{2}{\sqrt{3}}$, $\theta = \frac{\pi}{6}$, $z = 6$ in cylindrical coordinates.
   (c) The point $(x,y,z) = (\sqrt{2}, \sqrt{2}, 0)$ in Cartesian coordinates is the same as $\rho = 2$, $\varphi = \frac{\pi}{2}$, and $\theta = \frac{\pi}{4}$ in spherical coordinates.
   (d) The vector field $\mathbf{F}(x,y,z) = \langle 0,0,0 \rangle$ is conservative.
   (e) If $\mathbf{F}$ is a conservative field and C is parameterized by $\mathbf{r}(t)$, $a \leq t \leq b$, then $\int_C \mathbf{F} \cdot d\mathbf{r} = \mathbf{F}(\mathbf{r}(b)) - \mathbf{F}(\mathbf{r}(a))$.

2. (10 points)
   (a) Express the volume of the solid region, E, that sits above the rectangle in the xy-plane with vertices $(1,1,0)$, $(4,1,0)$, $(1,2,0)$, and $(4,2,0)$ and below the surface $z = xy$ in terms of a *double* integral.
   (b) Evaluate the integral you found in part (a).

3. (15 points) Let R be the region in the plane inside the circle $x^2 + y^2 = 2y$ and above the line $y = 1$.
   (a) Sketch R.
   (b) Express R in polar coordinates.
   (c) Compute $\iint_R f(x,y)dA$ for

$$f(x,y) = \frac{x}{\sqrt{x^2+y^2}}.$$

4. (10 points)
   (a) Rewrite the iterated integral

$$\int_0^2 \int_x^2 \int_0^{4-x-y} xy\,dz\,dy\,dx$$

   as an iterated integral in the order $dz\,dx\,dy$.
   (b) Evaluate *one* of the integrals from part (a).

The exam continues on the back!

5. (15 points) Let $f(x,y,z) = z$ and let W be the region that is above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 8$.

   (a) Setup (but *do not evaluate*) the integral $\iiint_W f(x,y,z)\,dV$ in both
      i. Cylindrical Coordinates, and
      ii. Spherical Coordinates.

   (b) Evaluate $\iiint_W f(x,y,z)\,dV$ using *one* of the integrals in part (a).

6. (10 points) Evaluate $\iiint_V \frac{y}{(x^2+y^2)^{1/2}}\,dV$, where

$$V = \{(r,\theta,z) \mid 0 \le r \le 6, 0 \le \theta \le \pi, 0 \le z \le 2/r\}.$$

7. (10 points) Evaluate the line integral

$$\int_C (3x^2 - 2y^2) ds$$

   where C is the curve parameterized by $\mathbf{r}(t) = (4t, 3t)$ for $0 \le t \le 2$.

8. (10 points) Let $f(x,y,z) = \cos(x)\sin(y)e^z$.

   (a) Compute the gradient field, $\mathbf{F} = \nabla f$.

   (b) Compute the integral, $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where C is the path given by $\mathbf{r}(t) = (\cos(2t), \sin(2t), \cos(t))$, $0 \le t \le 2\pi$.

9. (10 points) Let $\mathbf{F}(x,y,z) = (2xy + e^z, x^2 - \sin(y), xe^z + 7)$. Is $\mathbf{F}$ conservative? If so, find a potential function for $\mathbf{F}$.

End of Exam