Tufts University

Math 13 | **Department of Mathematics** | **October 17, 2011**

Exam 1 | **12:00 noon to 1:20 pm**

Instructions: No calculators, notes or books are allowed. Unless otherwise stated, you must show all work to receive full credit. **Simplify your answers as much as possible.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 points) **True or False - No Partial Credit**: On the first page of your blue book, answer the following questions as **True** or **False**.

   (a) $\mathbf{i} \times (\mathbf{j} \cdot \mathbf{k}) = 1$.

   (b) If $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ then $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$.

   (c) The function $f(x, y) = xy$ has no critical points.

   (d) The gradient of f at a point (a, b) is orthogonal to the level curve of f which contains (a, b).

   (e) The level curves of $z = f(x, y)$ for $f(x, y) = x^2 + y^2$ are parabolas.

2. (8 points) Sketch the curve defined by the vector function

$$\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle, 0 \leq t \leq 2\pi.$$

3. (12 points) An angry bird is moving along a path parameterized by $\mathbf{r}(t)$ with initial position given by

$$\mathbf{r}(0) = \langle -2, 0, 1 \rangle$$

   and velocity given by

$$\mathbf{v}(t) = \langle 4t \sin(t^2), 4t \cos(t^2), 2t \rangle \text{ for } t \geq 0.$$

   (a) Compute the acceleration, $\mathbf{a}(t)$, for the angry bird for any $t \geq 0$.

   (b) Compute the trajectory of the angry bird, i.e. its position vector for all $t \geq 0$, $\mathbf{r}(t)$.

   (c) Compute the arc length function for the vector-valued function measured from $t = 0$ and reparameterize the curve found in part (b), using this arc length function, $s(t)$.

The exam continues on the back!

4. (12 points) Consider the function $f(x, y) = \sqrt{y - x^2}$.

   (a) Find the domain and range of the function.

   (b) Sketch the surface corresponding to the square of this function, $z^2 = y - x^2$. To get full credit, you must show all of your work, including a few 2-dimensional sketches of the traces of the surface.

   (c) Give the name of the surface drawn in part (b).

5. (12 points) Find all first partial derivatives of $f(x, y) = x \ln(xy + 1)$

6. (12 points) Given the function $f(x, y, z) = x^3 - 2y^2 + 2z^2 - 4$,

   (a) Find the gradient of $f(x, y, z)$ at the point $P(1, -1, 3)$.

   (b) What is the direction of the maximum increase in the value of $f(x, y, z)$ at P?

   (c) What is the derivative of $f(x, y, z)$ in the direction of $\mathbf{u} = \langle \frac{4}{5}, \frac{3}{5}, 0 \rangle$ at P?

7. (10 points) Let $w(x, y, z) = \ln(x + y + z)$, where $x(s, t) = st$, $y(s, t) = s + t$, and $z(s, t) = s - t$. Find $\frac{\partial w}{\partial s}$, expressed as a function of s and t only.

8. (12 points) Given the function $f(x, y) = (x^2 - 4)(y + 3)$,

   (a) Find all critical points of $f(x, y)$

   (b) Use the Second Derivative Test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point.

9. (12 points) A rectangle with sides parallel to the $x-$ and $y-$axes is inscribed in the region in the first quadrant bounded by the axes and the line $x + 2y = 2$. Using Lagrange Multipliers, find the maximum area of this rectangle. No credit will be given for solutions that do not use Lagrange Multipliers.

End of Exam