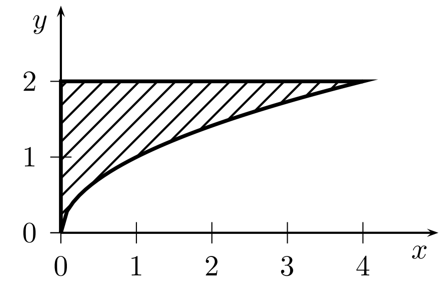

Figure 1: Domain for problem 3

Math 13 | Tufts University | November 15, 2010

Department of Mathematics

Exam 2

No calculators, notes, scratch paper or books are allowed. You must show all your work in your blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. **Simplify** *your answers as much as possible. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.*

1. (15 points) Find and classify (as local minima, local maxima, or saddle points) all critical points of the function $g(x,y) = x^4 - 2xy + \frac{1}{2}y^2$.

2. (15 points) Use the method of Lagrange multipliers to find the maximum volume of a rectangular box *without a top* that can be made of 12 square meters of material. *No credit will be given for a solution that does not use Lagrange multipliers!*

3. (10 points) Evaluate the integral $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{1+y^3}\, dy\, dx$ by reversing the order of integration.

4. (15 points) Figure 2 shows the solid bounded by the surfaces

$$x = 0, \quad y = z^2, \quad z = 2, \quad \text{and} \quad y = 2x$$

as well as the projection of the solid onto the xy-plane. **Set up** iterated *triple* integrals that yield the volume of the solid in the following orders. ***Do not evaluate!***

(a) $dz\ dy\ dx$

(b) $dx\ dy\ dz$

Exam Continues

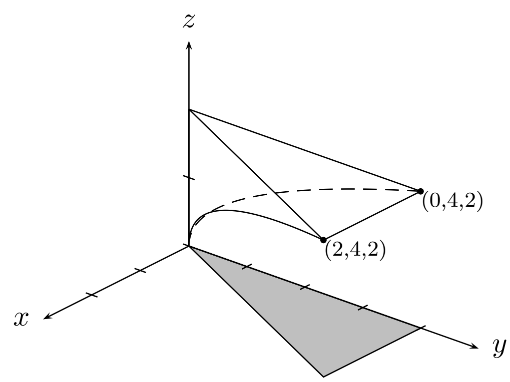

Figure 2: Problem 4

5. (15 points) Let E be the region in the first octant bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 32$. See Figure 3 below. Consider the integral

$$\iiint\limits_E xz \, dV.$$

(a) Rewrite the integral as an iterated integral in *rectangular coordinates.*

(b) Rewrite the integral as an iterated integral in *cylindrical coordinates.*

(c) Rewrite the integral as an iterated integral in *spherical coordinates.*

(*Note:* ***do not evaluate*** *any of the above.*)

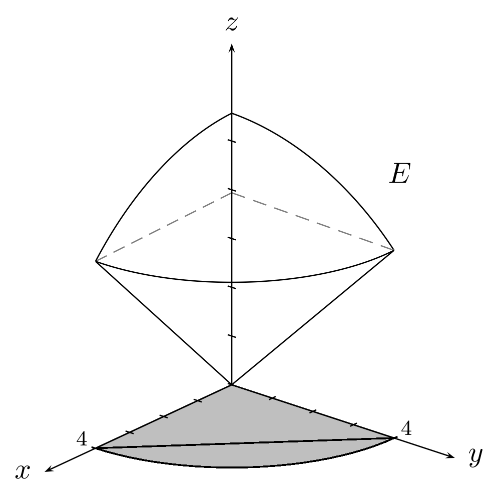

Figure 3: Problem 5

6. (15 points) Evaluate the integral $\displaystyle\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x \, dz \, dy \, dx.$

(*Hint: this may be easier in another coordinate system.*)

7. (15 points) Evaluate the following line integrals.

(a) $\int\limits_C xy^3 \, ds$ where C is the quarter circle of radius 2 shown in Figure 4.

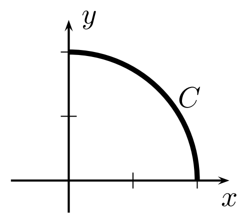

Figure 4: Problem 7a

(b) $\displaystyle\int_C \mathbf{F}(x,y,z)\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = (x+y)\,\mathbf{i} - x\,\mathbf{j} + \sin^2 x\,\mathbf{k}$ and C is the curve parametrized by $\mathbf{r}(t) = t\,\mathbf{i} - t^2\,\mathbf{j} + \cos^2 t\,\mathbf{k}$ for $0 \le t \le \dfrac{\pi}{2}$.

End of Exam