1. (15 points) Find and classify (as local minima, local maxima, or saddle points) all critical points of the function \( g(x, y) = x^4 - 2xy + \frac{1}{2}y^2 \).

2. (15 points) Use the method of Lagrange multipliers to find the maximum volume of a rectangular box without a top that can be made of 12 square meters of material. No credit will be given for a solution that does not use Lagrange multipliers!

3. (10 points) Evaluate the integral \( \int_0^4 \int_0^2 \frac{1}{1 + y^2} \, dy \, dx \) by reversing the order of integration.

4. (15 points) Figure 2 shows the solid bounded by the surfaces

\[ x = 0, \quad y = z^2, \quad z = 2, \quad \text{and} \quad y = 2x \]

as well as the projection of the solid onto the \( xy \)-plane. Set up iterated triple integrals that yield the volume of the solid in the following orders. Do not evaluate!

(a) \( dz \, dy \, dx \)

(b) \( dx \, dy \, dz \)
5. (15 points) Let $E$ be the region in the first octant bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 32$. See Figure 3 below. Consider the integral

$$\iiint_E xz \, dV.$$ 

(a) Rewrite the integral as an iterated integral in \textit{rectangular coordinates}.

(b) Rewrite the integral as an iterated integral in \textit{cylindrical coordinates}.

(c) Rewrite the integral as an iterated integral in \textit{spherical coordinates}.

(Note: \textit{do not evaluate} any of the above.)

6. (15 points) Evaluate the integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{9-x^2-y^2} x \, dz \, dy \, dx.$$  

(Hint: this may be easier in another coordinate system.)

7. (15 points) Evaluate the following line integrals.

(a) $\int_C xy^3 \, ds$ where $C$ is the quarter circle of radius 2 shown in Figure 4.
(b) \[ \int_C \mathbf{F}(x, y, z) \cdot d\mathbf{r} \] where \( \mathbf{F}(x, y, z) = (x+y) \mathbf{i} - x \mathbf{j} + \sin^2 x \mathbf{k} \) and \( C \) is the curve parametrized by \( \mathbf{r}(t) = t \mathbf{i} - t^2 \mathbf{j} + \cos^2 t \mathbf{k} \) for \( 0 \leq t \leq \frac{\pi}{2} \).