Department of Mathematics
Exam 1

No calculators, notes, scratch paper or books are allowed. You must show all your work in your blue book in order to receive full credit. A correct answer with no work may not necessarily score any points. **Simplify** *your answers as much as possible. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.*

1. (15 points): Consider the triangle whose vertices are

$$P(1,2,0) \quad Q(2,1,2) \quad R(4,2,1).$$

   (a) Give parametric equations for the line containing the edge PQ.

   (b) Give an equation for the plane containing the triangle $\triangle PQR$.

   (c) Find the area of the triangle $\triangle PQR$.

2. (15 points): Let

$$\vec{u} = \vec{\imath} + \vec{\jmath} - \vec{k}, \quad \vec{v} = 2\vec{\imath} - \vec{\jmath} + 3\vec{k}.$$

   Calculate

   (a) The cosine of the angle between $\vec{u}$ and $\vec{v}$.

   (b) $\text{comp}_{\vec{v}}\,\vec{u}$, the scalar projection of $\vec{u}$ onto $\vec{v}$;

   (c) $\text{proj}_{\vec{u}}\,\vec{v}$, the vector projection of $\vec{v}$ onto $\vec{u}$ *(Note that the roles of the two vectors have been switched from the previous question.)*;

3. (10 points): Let

$$f(x,y) = x^2 - y.$$

   (a) Sketch and label the level curves of f corresponding to the values $f = -1, 0$, and 1.

   (b) Sketch the trace of the graph $z = f(x,y)$ in the (x,z) plane, labeling the x and z axes.

   (c) Sketch the trace of the graph $z = f(x,y)$ in the (y,z) plane, labeling the y and z axes.

4. (15 points): A particle moves with acceleration

$$\vec{a}(t) = (t+1)\vec{\imath} + e^{2t}\vec{\jmath} + \pi \sin \pi t \vec{k}$$

initial velocity

$$\vec{v}(0) = \vec{\imath} + \vec{\jmath}$$

and initial position

$$\vec{r}(0) = \vec{\imath} + \vec{\jmath} + \vec{k}.$$

(a) Give a formula for its position at any later time t (that is, find the position function).

(b) Suppose w is related to the position of the particle via

$$w = \frac{x^2 z^3}{y}.$$

Find $\frac{dw}{dt}$ when $t = 0$.

5. (5 points): Consider the lines given parametrically by:

$$L_1 : \begin{cases} x = 1 + 2t \\ y = -1 + 3t \\ z = 2 - t \end{cases} \qquad L_2 : \begin{cases} x = -1 + t \\ y = -4 + 2t \\ z = 3 + 3t \end{cases}$$

Either show that these lines don't intersect, or give the coordinates of a point of intersection.

6. (10 points): Give parametric equations for the line of intersection of the two planes

$$2x - y + 3z = 4 \quad \text{and} \quad x - 2y + z = 0.$$

7. (10 points): Consider the function

$$f(x,y) = \sqrt{x^2 + xy + 2y^2}.$$

Use the linear approximation at $(2, -3)$ to calculate an approximate value for $f(2.2, -3.1)$.

8. (10 points): Let

$$f(x,y,z) = x^2 e^{2y+z}.$$

(a) Calculate the gradient vector $\nabla f(3,1,-2)$ of f at $(3,1,-2)$.

(b) Give an equation for the plane tangent to the surface

$$x^2 e^{2y+z} = 9$$

at the point $(3,1,-2)$.

9. (10 points): If x, y, and z are related by

$$z^4 + x^2 yz + 2xy = 1,$$

find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $x = -1$, $y = 2$, and $z = 3$.