Tufts University

Math 13 | Department of Mathematics | December 16, 2009

Final Examination | 8:30-10:30 a.m.

Instructions: Please do all problems below. You must show your work and justify your answers in order for partial or full credit to be awarded. No books, notes or calculators are allowed during this exam. *You are required to sign each examination blue book that you are handing in. With your signature, you are pledging that you have neither given nor received any help pertaining to this exam. If you are found in violation of this policy, you will be refered to the Dean of Students and automatically receive an* **F** *for the course.*

1. (10 points):

   (a) Give an equation for the plane containing the points $(2,2,0)$, $(3,0,1)$, and $(6,0,0)$.

   (b) Give an equation for the plane tangent to the surface

   $$x^2y - 2xyz + 3z^2 = 3$$

   at the point $(-2,-3,-1)$.

2. (10 points): Find the minimum and maximum values of the function $f(x,y,z) = 8x - 4z$ on the ellipsoid $x^2 + 10y^2 + z^2 = 5$.

3. (10 points):

   (a) Find all the critical points of the function $f(x,y) = x^4 + 32xy + 32y^2$. (Caution: note that the exponents in the first and third terms are different.)

   (b) The function $f(x,y) = 8x^3 + y^3 + 6xy$ has a critical point at $(-\frac{1}{2}, -1)$. Determine whether it is a local minimum, a local maximum, or neither, for $f(x,y)$.

4. (10 points): Evaluate the line integral $\int_{\mathcal{C}} y\, ds$, where $\mathcal{C}$ is the curve given by the parametric equations

   $$\begin{cases} x = 3t - t^3 \\ y = 3t^2 \end{cases}, \quad -\sqrt{3} \le t \le \sqrt{3}.$$

Exam continues on back

5. (10 points): For each region set up, but **do not evaluate**, a triple integral in the specified coordinates expressing the given volume:

   (a) The volume of the region *in the first octant* outside the cylinder $x^2 + y^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 4$ (see Fig 1a), in **cylindrical coordinates**.

   (b) The volume of the region *in the first octant* inside the sphere $x^2+y^2+z^2 = 2$ and outside the cone obtained by rotating about the z-axis a ray making angle $\frac{\pi}{6}$ radians with the positive z-axis (see Fig 1b), in **spherical coordinates**

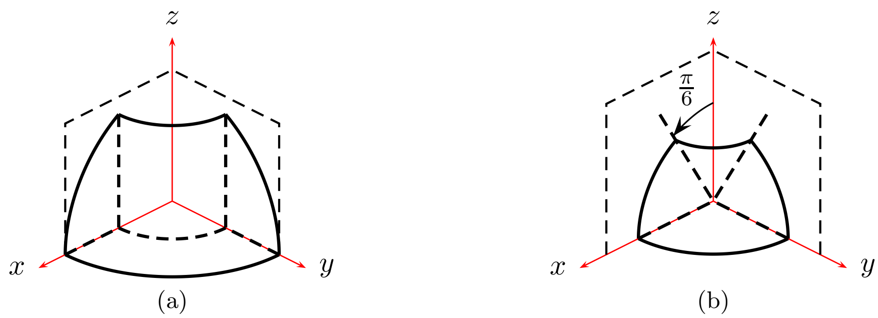

Figure 1: Problem 5

6. (10 points): Let $\mathcal{E}$ be the pyramidal region with vertices $(0, 0, 0)$, $(1, 0, 1)$, $(0, 1, 1)$, and $(0, 0, 1)$ (see Fig 2).

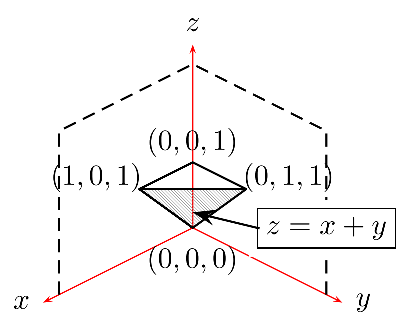

Figure 2: Problem 6

Express the integral

$$\iiint_{\mathcal{E}} f(x, y, z) \ dV$$

as an iterated integral in the order

   (a) $dz \ dy \ dx$;

   (b) $dy \ dz \ dx$.

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7. (10 points): Calculate the surface integral (flux integral)

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S} = \iint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{n} \, dS$$

of the vector field

$$\mathbf{F}(x, y, z) = -x\mathbf{i} - y\mathbf{j} + z\mathbf{k}$$

over the surface $\mathcal{S}$ given by $z = 4 - x^2 - y^2$, $z \geq 0$.

8. (15 points): Calculate the surface integral

$$\iint_{\mathcal{S}} x^2 z^2 \, dS$$

where $\mathcal{S}$ is the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 1$ and $z = 2$ (see Fig 3).

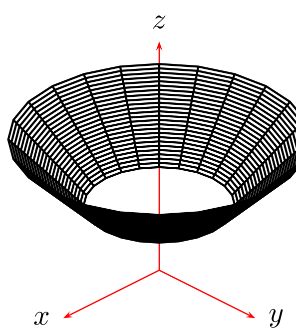

Figure 3: Problem 8

9. (15 points): Use the Divergence Theorem to calculate the flux of the vector field $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$ over the surface of the prism in the first quadrant bounded by the coordinate planes, the plane $x + y = 1$, and the plane $z = 1$ (see Fig 4), with outward orientation.

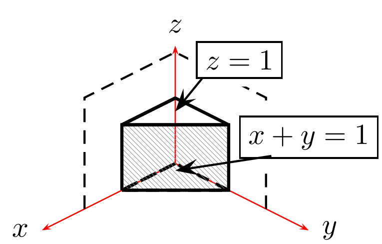

Figure 4: Problem 9

End of Exam.