Math 13 | Tufts University
Department of Mathematics
Exam 2 | November 23, 2009
Noon–1:20 p.m.

Instructions: Please do all eight problems below. You must show your work and justify your answers in order for partial or full credit to be awarded. No books, notes or calculators are allowed during this exam. *You are required to sign each examination blue book that you are handing in. With your signature, you are pledging that you have neither given nor received any help pertaining to this exam. If you are found in violation of this policy, you will be refered to the Dean of Students and automatically receive an* **F** *for the course.*

1. (15 points) Use Lagrange multipliers to find the maximum volume of a rectangular box in the first octant, with sides parallel to the coordinate axes, which has a vertex in the plane $x + 2y + 3z = 6$.

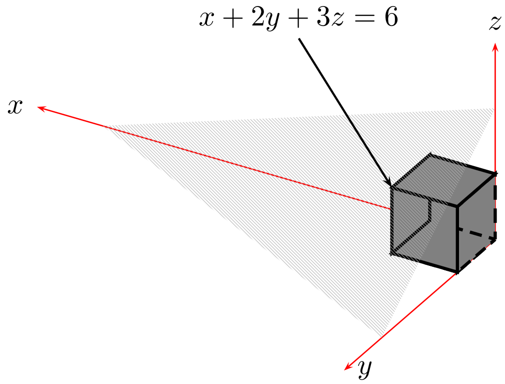

Figure 1: Problem 1

2. (13 points) Use spherical coordinates to find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$, below the cone $z = \sqrt{x^2 + y^2}$, and above the xy-plane.

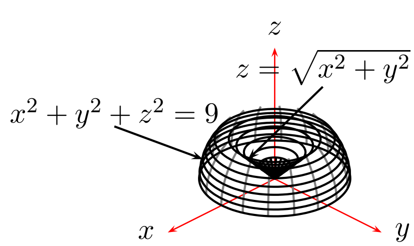

Figure 2: Problem 2

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3. (12 points) Let E be the solid in the first octant bounded by the parabolic cylinder $z = 1 - y^2$ and by the planes $y = x$ and $z = 0$.

   Express the triple integral $\iiint_E f(x, y, z)\, dV$ as an iterated triple integral

   (a) in the order $dz\, dy\, dx$

   (b) in the order $dx\, dz\, dy$.

   DO NOT EVALUATE.

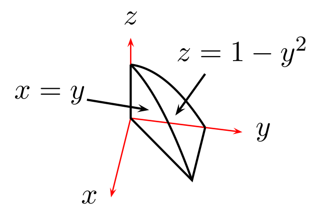

Figure 3: Problem 3

4. (10 points) Express the following iterated triple integral

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} (x^2 + y^2)^{\frac{3}{2}}\, dz\, dy\, dx$$

   in cylindrical coordinates. **DO NOT EVALUATE.**

5. (12 points) Evaluate $\oint_C y^3\, dx - x^3\, dy$, where C is the closed curve in the xy-plane consisting of the line segment from $(-1, 0)$ to $(1, 0)$ followed by semicircle $y = \sqrt{1 - x^2}$ from $(1, 0)$ back to $(-1, 0)$.

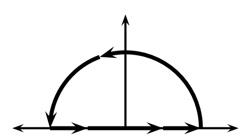

Figure 4: Problem 5

6. (10 points) Let S be the surface given by the vector valued function $\mathbf{r}(u, v) = (u^2 + v^2)\,\mathbf{i} + uv\,\mathbf{j} + u\,\mathbf{k}$. Let P be the point $(5, 2, 1)$ on S, corresponding to $u = 1$ and $v = 2$.

   (a) Find a normal vector to S at the point P.

   (b) Find an equation of the tangent plane to S at the point P.

7. (16 points)

(a) Evaluate the line integral

$$\int_C x^4\, y\, ds$$

where C is the part of the unit circle $x^2 + y^2 = 1$ in the first quadrant.

(b) Let $\mathbf{F}(x, y) = e^x\ \mathbf{i} + e^y\, \mathbf{j}$. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ on the curve C given by $x = \ln t,\ y = \ln\, 2t$, for $1 \le t \le 4$.

8. (12 points) Let $\mathbf{F}(x, y, z) = (\cos x + 2yz)\, \mathbf{i} + 2xz\, \mathbf{j} + (z + 2xy)\, \mathbf{k}$.

(a) Find a scalar function $f(x, y, z)$ such that $\mathbf{F} = \nabla f$.

(b) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the line segment from $(0, 0, 0)$ to (π, π, π).

End of Exam.