Tufts University

Math 13	**Department of Mathematics**	**October 19, 2009**
	Exam 1	**12:00 noon to 1:20 pm**

Instructions: No calculators, notes or books are allowed. You should show all work to receive full credit. **Simplify your answers.** Please circle your answers and cross out any work you do not want graded. *You are required to sign your exam book. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

1. (10 pts) **True/False – No Partial Credit** : On the inside cover of your blue book answer the following questions as either **True** or **False**.

   a) $\vec{i} \cdot \vec{j} = \vec{k}$.

   b) The equation $z^2 - 1 = x^2 + y^2$ is that of a cone.

   c) The line $\vec{r}(t) = 2\vec{i} + (1+t)\vec{j} - 5\vec{k}$ is orthogonal to the plane $y = -3$.

   d) Two vectors $\vec{u}$ and $\vec{v}$ are orthogonal if and only if $\vec{u} \times \vec{v} = \vec{0}$.

   e) If (x_0, y_0) is a critical point of the function $z = f(x, y)$, then f has either a local maximum or a local minimum at (x_0, y_0).

2. (10 pts)

   a) Find an equation of the plane that contains the lines $\vec{r_1}(t) = \langle 1, 2, 3\rangle + t\langle 2, -1, 2\rangle$ and $\vec{r_2}(s) = \langle 1, 2, 3\rangle + s\langle 3, 1, 1\rangle$.

   b) Find parametric equations for the line that passes through the point $(1, 2, 6)$ and is orthogonal to the plane $2x - 3y + z = 4$.

3. (10 pts) Let $y = x^2 + z^2$. Draw four separate sketches, as described below:

   a) Sketch the trace in the plane $x = 0$.

   b) Sketch the trace in the plane $y = 1$.

   c) Sketch the trace in the plane $z = 0$.

   d) Sketch the surface given by this equation.

4. (10 pts) A particle is moving along the helical path $\vec{r}(t) = (\cos 3t)\vec{i} + (\sin 3t)\vec{j} + (4t)\vec{k}$.

   a) Find parametric equations for the line tangent to this curve when $t = \frac{\pi}{4}$.

   b) Find the distance traveled from time $t = 0$ to $t = 2$.

5. (15 pts) Let $f(x, y) = \sqrt{x^2 + 4y^2}$.

   a) Find the linearization, $L(x, y)$, of f at $(3, 2)$.

   b) Use $L(x, y)$ (or the linear approximation) to estimate $f(3.1, 1.9)$; give your answer as a decimal expression to two places.

6. (10 pts) Consider the surface S given by the equation $x^2y + yz^2 - e^{xz} = 3$. Find an equation of the plane tangent to S at the point $(2, 1, 0)$.

   Exam continues on the back!

7. (10 pts) Consider the function $f(x, y) = 3x^2 + 4y^2$.

   a) Find a unit vector in the direction in which f is increasing the fastest at the point $(1, 1)$.
   b) Find the directional derivative of f in the direction of the vector $\vec{v} = \langle 4, 3\rangle$ at the point $(1, 1)$.

8. (10 pts) The function $f(x, y) = x^3 - 3xy + \frac{1}{2}y^2$ has critical points $(0, 0)$ and $(3, 9)$. For each of these two points, determine if it is a local maximum, a local minimum or a saddle point.

9. (15 pts) Let $f(x, y) = \frac{1}{3}x^3 + y^2$. Find the absolute maximum and the absolute minimum values of f on the unit disk $D = \{(x, y)|x^2 + y^2 \leq 1\}$. (see figure 1 below)

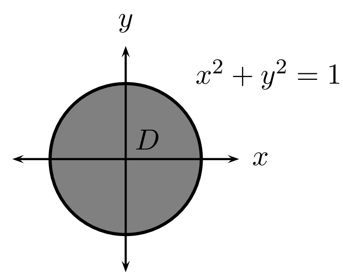

Figure 1: Problem 9

End of Exam