Math 13	Tufts University Final Examination (100 points)	Bluebook No. ____________ December 12, 2008 8:30 a.m. – 10:30 a.m.

You are not allowed to use books, notes, or electronic devices such as calculators, cellphones, or personal digital assistants during this exam. Please sign your exam book; in so doing you are certifying that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

You should show all work to receive full credit; please make sure that your answer (and reasoning) can be understood, from what you have written, without undue effort. This exam consists of **TEN** *problems; do not forget to do the last problem, which is on Page 4.*

1. (a) (5 points) Let L be the line with parametric equations $x = 2 + t,\ y = 2 + 3t,\ z = -7t$ and P the point $(-13, 0, 1)$. Find an equation for the plane containing the point P and perpendicular to the line L.
   (b) (5 points) The largest directional derivative of $f(x, y, z)$ at the point $(1, 0, 5)$ occurs in the direction of the unit vector $\vec{u} = \frac{1}{3}(\vec{\imath} + 2\vec{\jmath} + 2\vec{k})$. If $f_x(1, 0, 5) = 3$ what are the values of $f_y(1, 0, 5)$ and $f_z(1, 0, 5)$?

2. (10 points) Let $f(x, y) = xy + x^2 + y^2$ and let D be the unit disk $D = \{(x, y) | x^2 + y^2 \leq 1\}$. Find the absolute maximum and the absolute minimum values of $f(x, y)$ on D and the points at which these values are attained.

3. (a) (5 points) Write down an integral in cylindrical coordinates for the volume of the solid enclosed between the two paraboloids $z = x^2 + y^2$ and $z = 2 - (x^2 + y^2)$. **DO NOT EVALUATE THE INTEGRAL**.
   (b) (5 points) Change the order of integration in the following integral. **DO NOT EVALUATE THE INTEGRAL**.

$$\int_0^2 \int_{x^2}^{\sqrt{8x}} x^2 y \, dy dx$$

4. (10 points) An unidentified object is moving with acceleration $\vec{a}(t) = e^{-t}\vec{\imath} + \vec{k}$, where t is measured in seconds and the acceleration in ft/s^2 and where the x-axis points east, the y-axis north, and the z axis upward. When first observed ($t = 0$), the object was 300 ft directly above the Los Alamos observatory and moving north with velocity 50 ft/s. Write down the position vector of the object 10 seconds later.

5. (10 points) Let $\vec{F} = yz\vec{\imath} + (xz + 3y)\vec{\jmath} + (xy + 5)\vec{k}$
   (a) Show that $\vec{F}$ is conservative.
   (b) Find a potential function $f(x, y, z)$ for $\vec{F}$.
   (c) Compute $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ where $\mathcal{C}$ is the perimeter of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ in the xy-plane traversed once counterclockwise starting and ending at the point $(2, 0, 0)$. (See Figure 1 on next page.)

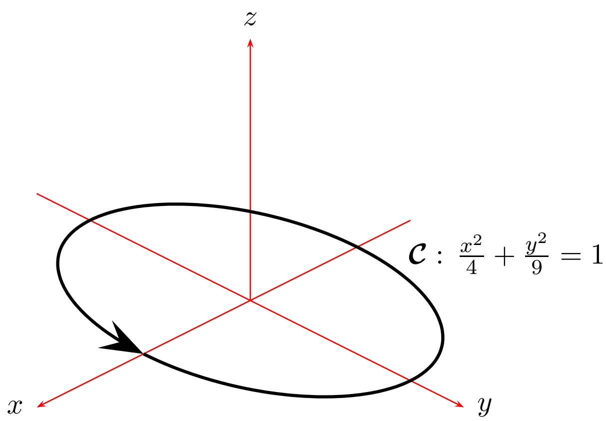

FIGURE 1. Problem 5c

6. (10 points) Use Green's Theorem to evaluate the integral

$$\oint_{\mathcal{C}} (y + e^{x^2})\, dx + (2x + \ln(y^2))\, dy$$

where $\mathcal{C}$ is the perimeter of the rectangle with vertices $(0,0), (3,0), (3,2), (0,2)$ oriented counterclockwise.

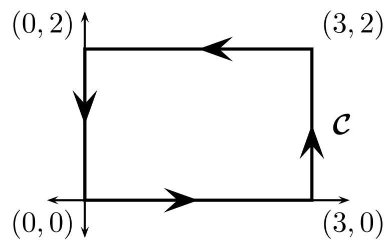

FIGURE 2. Problem 6

7. (10 points) Let $\mathcal{S}$ be the portion of the cone $3(x^2 + y^2) = z^2$ that lies above the xy-plane and below the sphere $x^2 + y^2 + z^2 = 9$. Find a parametric representation for the surface $\mathcal{S}$; be sure to include the limits for the parameters. (Note that $\mathcal{S}$ is a surface, not a solid cone.)

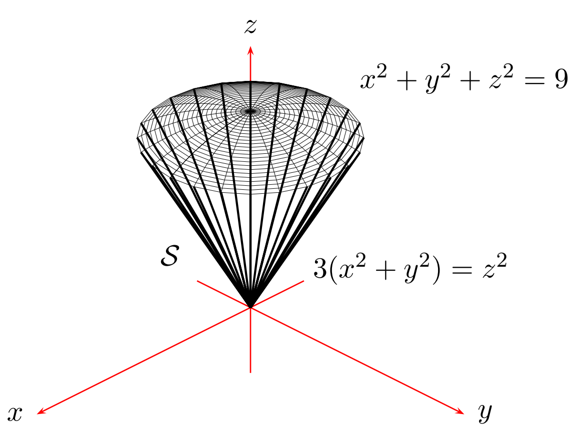

FIGURE 3. Problem 7

8. (10 points) Let $\mathcal{S}$ be the part of the saddle-shaped surface $x^2 - y^2 = z$ inside the cylinder $x^2+y^2 = 4$ and let $\vec{F}$ be the vector field $x\vec{\imath}-y\vec{\jmath}+\vec{k}$. Compute the surface integral $\iint_{\mathcal{S}} \vec{F}\cdot d\vec{S}$ with respect to the upward unit normal.

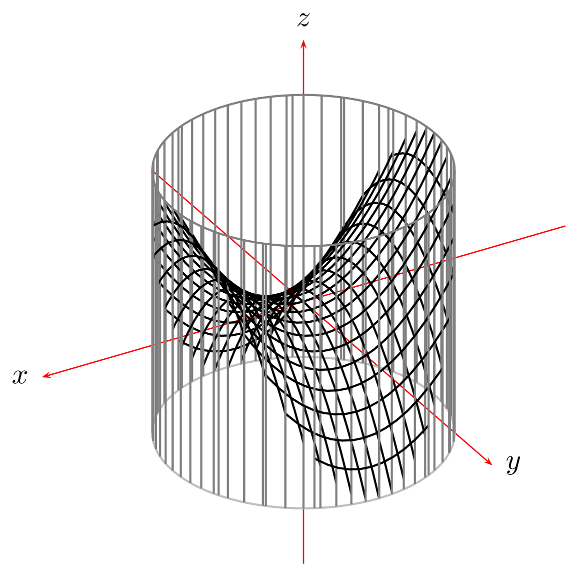

Figure 4. Problem 8

9. (10 points) Consider the vector field $\vec{F} = x^2y\vec{\imath}+2xz\vec{\jmath}+yz^3\vec{k}$. Use the Divergence Theorem to find the outward flux of $\vec{F}$ across the surface of the rectangular box with vertices

$$(0,0,0), (1,0,0), (0,1,0), (1,1,0), (0,0,5), (1,0,5), (0,1,5), (1,1,5).$$

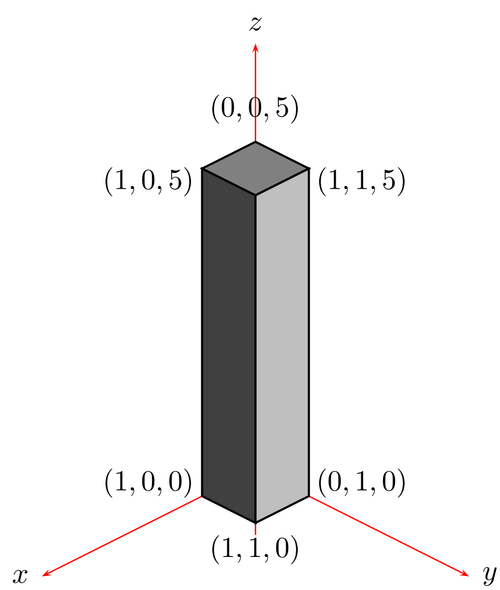

Figure 5. Problem 9

(Turn over for Problem 10.)

10. (10 points) Consider the vector field $\vec{F} = (y - x + \sin(x^3 + 3))\vec{\imath} + (x - z + \ln(y^2))\vec{\jmath} + (x^2 - y)\vec{k}$. Use Stokes' Theorem to compute $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where $\mathcal{C}$ is the triangle obtained as the intersection of the plane $x + 2y + z = 2$ with the first octant oriented counterclockwise viewed from above.

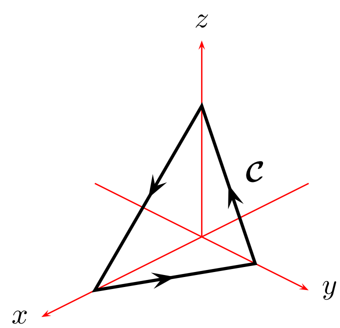

FIGURE 6. Problem 10