Name:____

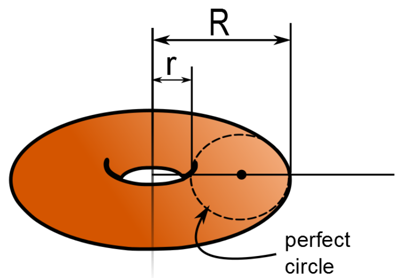
Professors name:____

Due date: 12/4

1. It's 1948 and William Rosenberg is perfecting his doughnut receipe for the grand opening of The Open Kettle in Quincy, MA[1]. The Rosenberg family makes delicious doughnuts with perfectly circular cross sections (no easy feat)! The inner radius of the doughnut is r and the outer radius is R (see picture below).

   (a) Compute the volume of a doughnut in terms of r and R.
   Hint: You'll need some washers and trig subs to get through this problem!

   (b) Rosenberg decides to increase the volume of the doughnuts. Which gives larger doughnuts, increasing the outer radius by 10% or decreasing the inner radius by 10%? Your argument should be backed up with calculations.



2. An eagle is standing on the edge of a river, located at $x = 1$. It takes off and flies in a path given by $y = \ln(x)$ starting at $x = 1$.

   (a) Find the distance the eagle traveled along the path from $x = 1$ to $x = a$ and call it $L(a)$.
   Hint: A clever u-substitution of $u = \sqrt{x^2 + 1}$ will make the problem work out nicer.

   (b) As a increases, $L(a)$ increases. Find the power n such that $L(a) \sim a^n$ for large a.

3. Compute $\displaystyle\int_{-\infty}^{\infty} \frac{2x^3 + 2x^2 + 4x + 4}{x^2(x^2 + 4)}\, dx$. You must show ALL work, or fully explain why you skipped any steps.

4. Evaluate the following improper integrals with the help of L'Hopital's Rule:

   (a) $\displaystyle\int_0^a x^x(\ln x + 1)\, dx, \quad a > 0.$

   (b) $\displaystyle\int_0^\infty x^{-x}(\ln x + 1)\, dx$

[1] Yes, this is the origin story of Dunkin Donuts. Read more on the history of the doughnut here!