

Name: _____

Professors name: _____

Due date: 10/30

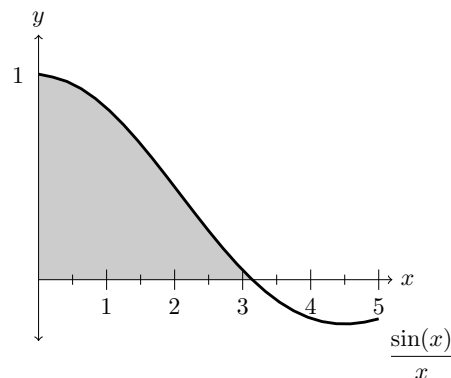
1. In this problem, we estimate the area under $\frac{\sin(x)}{x}$ between 0 and π .

(a) Find a power series representation for $\frac{\sin(x)}{x}$. Use a Taylor Series that you know and what you know about combining power series.

(b) What is the interval of convergence for this power series?

(c) Estimate $\int_0^\pi \frac{\sin(x)}{x} dx$ using six terms of the associated power series.

(d) What is a bound on the error of your estimation in (c)?



FUN FACT: A mathematical mystery of antiquity is the Basel problem: we know $\sum_{k=1}^{\infty} \frac{1}{k^2}$ is a convergent p -series, but what is the limit? Euler solved this problem in 1734 using the Taylor series representation for $\frac{\sin(x)}{x}$. Use online resources to check out Euler's proof of the Basel problem for finding out the exact limit of $\sum_{k=1}^{\infty} \frac{1}{k^2}$. The proof shows that it's not always easy to find the limit of a convergent series!

2. Cicadas have an oscillating population growth with prime life cycles. The population growth rate of cicadas is

$$N'(t) = \frac{100\pi}{13} \sin\left(\frac{2\pi t}{13}\right)$$

(a) Suppose you count 10 screaming cicadas in your yard at initial time $t = 0$ years. Find $N(t)$.

(b) When do cicadas reach their first peak? What is the peak number of cicadas? How long does it take for the cicadas to reach their next peak?

(c) Cicada killer wasps (yes, that's a thing) eat cicadas. Consider the population growth rates of killer wasps:

$$W'(t) = -\frac{80\pi}{3} \cos\left(\frac{2\pi t}{3}\right)$$

Find the formula for $W(t)$ with initial population of $W(0) = 75$.

(d) Graph $N(t)$ and $W(t)$ on the same plot. Is it likely that the cicadas and their predators will ever peak around the same time?